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A NONLINEAR PROGRAMMING MODEL OF A WASTEWATER TREATMENT SYSTEM: SENSITIVITY ANALYSIS AND A ROBUSTNESS CONSTRAINT

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ABSTRACT

A method for sensitivity analysis in nonlinear programming is described and then illustrated using a least-cost model of a secondary wastewater treatment system. A sensitivity equation approach is used to calculate normalized sensitivity coefficients, which approximate the percent changes in model variables and objective function due to a small parameter variation. Design changes predicted by the sensitivity coefficients are confirmed by a perturbation analysis of the optimal solution. Sensitivity concepts are used to develop a robustness measure which is incorporated into the constraint set of the nonlinear model. Robustness is narrowly defined as the ability of a model solution to maintain a level of performance that meets the system design criteria even if the actual values of model parameters are not exactly the same as the values assumed for design. A gradient optimization procedure is used to examine the tradeoff between total cost and the robustness measure. A preliminary analysis shows that the trends in robust wastewater treatment plant design are in direct conflict with the optimal decisions obtained when minimizing cost without a constraint on robustness but are in agreement with those designs observed to work in practice. The robustness constraint method presented should be applicable to other optimization models of water resources systems.

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CHAPTER 1

INTRODUCTION

This research uses a mathematical wastewater treatment plant model, developed by Tang [1984], to illustrate a method of sensitivity analysis in nonlinear programming (NLP). A sensitivity equation approach is used to calculate normalized sensitivity coefficients, which approximate the percent changes in model variables and objective function due to a small parameter variation. Design changes predicted by the sensitivity coefficients are confirmed by a perturbation analysis of the optimal solution.

In addition, the wastewater treatment plant model is used in a preliminary analysis of a method for incorporating a *robustness* measure that is based on the notion of system sensitivity into nonlinear optimization models. Robustness is defined for this research as the ability of a model solution to maintain a level of performance that meets the system design criteria even if the actual values of model parameters are not exactly the same as the values assumed for design.

The "optimal" design is presented and discussed throughout this report. However, the true optimal solution cannot be found through use of the mathematical model alone, because:

[1] The optimization of systems described by highly nonlinear mathematical equations, such as those predicting the performance of various unit processes of a wastewater treatment facility, is at best difficult and, in general, proving that a particular solution satisfies even the mathematical necessary conditions for optimality may not be possible. Also, for a very nonlinear process with many interactions a multitude of local optima are expected to exist.

[2] Models never simulate reality exactly but usually strive to capture the significant interactions within a process. Thus the results of the analysis will not, in general, mirror precisely the real world.

[3] Usually the modeler must choose one objective to be minimized or maximized. Real design is always multiobjective, and since these objectives usually conflict, the true optimal design will be less than optimal with respect to any one single objective. Even if multiobjective optimization techniques are employed, a decision maker is required to arrive at the final design.

[4] The wastewater treatment plant considered in this research is not real. The technological parameters, including influent conditions, were arbitrarily selected to represent average values found in practice. Therefor, no ranking of the parameters by their significance to the design can be made; nothing is known of the uncertainty level or uncertainty distribution of the parameters since the values are hypothetical.

For these reasons, it is not the objective of this work to present the sensitivity of the "optimal" design. This research presents and evaluates a method of sensitivity analysis in nonlinear programming which is general for problems described by algebraic equations. The sensitivity information is shown to aid in examining the complex interactions present in the wastewater treatment plant model, and to help identify where potential cost savings lie. A designer of wastewater treatment facilities may find the information and discussion useful in choosing the best design for a particular situation, taking into account his or her experience and the concerns of others.

CHAPTER 2

LITERATURE REVIEW

Examples in the literature of sensitivity analysis applied to wastewater treatment plant design are not numerous. The research done can be roughly divided into two catagories based on the type of sensitivity analysis performed: perturbation or the sensitivity equation approach. Perturbation analyses are far more common; no research applying sensitivity equations to a realistic system was found. Below a review of relevant research conducted by the civil and chemical engineering communities is presented.

McBeath and Eliassen [1966] present a sensitivity analysis of a model of the activated sludge process. The final clarifier efficiency is determined from a simple mass balance around the aeration tank and settler, and the portion of biomass wasted contributes to a digester load factor. The emphasis of their research was not the development of the model but an illustration of the use and significance of sensitivity analysis in engineering design. No formal optimization procedure was employed; the values of all variables and parameters were assumed and the system cost was measured as these values were varied throughout a feasible range. The authors acknowledge the implied definition of sensitivity as a rate of change, but assert that sensitivity analysis must include the response of the system for the entire feasible range of the parameter in question, not just the slope of the response function at a point. Hence, McBeath and Eliassen did not develop sensitivity coeficients in their analysis.

The cost of the system was defined as being sensitive to a parameter or variable if it varied more than 10 percent when the parameter or variable value was perturbed to a high or low value. Using this definition, the annual cost was found to be sensitive to: the price of phosphorous additive, the constant in the expression for maintenance costs, the influent soluble 5-day biochemical oxygen demand (BOD_5), the plant design life, the time discount rate, the wastewater flow rate, the logarithmic BOD_5 removal rate, the treatability limit, the phosphorous use ratio, and the mixed liquor suspended solids concentration (MLSS) in the aeration tank. Also, the digester load factor was found to be sensitive to the sludge age and the concentration of influent soluble BOD_5. McBeath and Eliassen also point out that a simultaneous variation in two or more parameters may result in a large change in the response function, even though the same parameters varied individually produce little change. Finally, although their research did not investigate the optimal design, McBeath and Eliassen conclude that their "procedure offers a means of parameter study for the purpose of describing some characteristics of optimal areas available in the system."

Berthouex and Polkowski [1970] describe the optimization of an activated sludge wastewater treatment plant when parameter uncertainty is incorporated into the process models. The method of propagation of variance was used to predict the response of the system to variations in the parameters. This research incorporated the final settling tank as an integral part of the activated sludge system design. Other processes modeled include: primary sedimentation, anaerobic digestion, and sludge disposal. Total suspended solids were assumed not to contribute to the effluent BOD_5 since perfect final clarification was assumed. Also, the design of the solids handling system was not included in the optimization procedure. The Hooke-Jeeves search algorithm was used to find the least cost design of the plant under uncertainty.

The authors found that as the influent BOD_5 concentration increased, the MLSS concentration and the aeration tank volume increased because of an increase in the recycle ratio. However, this solution was quite sensitive to variations in parameters describing the activated sludge thickening characteristics. This supports inclusion of the final settler as an integral part of the activated sludge design.

The design was very sensitive to changes in the BOD_5 removal rate constant, k. When k was low, more emphasis was placed on primary sedimentation, and the aeration basin volume and MLSS level were increased. The increase in aeration tank volume was great relative to the increase in MLSS concentration. The influent soluble BOD_5 concentration also influenced the design and system cost significantly.

Incorporating uncertainty into the design increased the total cost approximately 7 to 12 percent over that of the solution when uncertainty was not considered. The effect was to increase the area of the primary settler and the MLSS concentration while the activated sludge design was otherwise unaffected. The loading rates on the aeration tank were lower when optimizing under uncertainty. This analysis illustrates the usefulness of incorporating uncertainty for estimating quantitative safety factors for plant design.

The propagation of uncertainty in k, the BOD_5 removal rate constant, varied with its magnitude, and with the MLSS concentration, the volume of the aeration tank, and the wastewater flow into the aeration basin. The design variables were found to be insensitive to variations in the power cost, oxygen transfer efficiency, and pumping head. Sensitivity coefficients were not calculated from their perturbation analysis.

Tarrer et al. [1976] expanded on the work of Berthouex and Polkowski by including a model to predict clarification efficiency of the activated sludge. Thus, effluent BOD_5 was comprised of suspended material and soluble material. However, like Berthouex and Polkowski, Tarrer et al. assumed the liquid and solids handling portions of the wastewater treatment plant could be optimized separately; the cost of sludge digestion and disposal was added to the cost of the liquid subsystem after determining the optimal liquid subsystem design. The Golden Section Search algorithm was used to optimize a system with two degrees of freedom.

Tarrer et al. included two constraints in their formulation which should be noted because of their effect on the least cost design. The sludge age was constrained to be at least four days and the MLSS concentration was required to be at least 2000 mg/l. The above constraints produced a design which was thought to be very conservative, particularly with respect to liquid-solid separation in the final clarifier. The overflow rates in both primary and final settlers were very low, as were the solids loadings applied to them. The low overflow rate of the final clarifier may be because of the constraints on sludge age and MLSS concentration and the use of a model to predict effluent *TSS*.

A sensitivity analysis was performed on the model. The values of the maximum specific substrate utilization rate, k, and the half velocity constant, K_s, were perturbed to simulate a hard to degrade wastewater. The sludge age was found to be very sensitive to these variations, while the total system cost increased only moderately. The increase in sludge age was achieved primarily through a larger aeration tank rather than an increase in MLSS concentration, although the MLSS concentration did increase enough to warrant an increase in the final settler area. Thus, a hard to degrade wastewater was observed to alter the sizes of the unit processes significantly but not to alter the optimal annual cost significantly.

The MLSS concentration, aeration tank volume, and the area of the primary clarifier were sensitive to variations in the activated sludge thickening characteristics. Poor thickening characteristics reduced the MLSS value, increased the aeration tank volume, and increased the area of the primary clarifier. The opposite was true of an activated sludge with good thickening properties. The total system cost was not sensitive to the above parameter variations.

The optimal design was very sensitive to the influent soluble BOD_5 concentration, as was the total cost. However, the changes in design variables were dependent on the MLSS concentration lower bound and the sludge age lower bound. For example, decreasing the influent soluble BOD_5 increased the area of the final settler because this was the most economical way to satisfy the MLSS constraint. The system design and cost were very insensitive to changes in the *TSS* effluent standard. No sensitivity coefficients were calculated from their analysis.

Tarrer et al. point out that sensitivity analysis is useful in determining which way parameters should be perturbed to obtain a more conservative design. Also, they recognized that results of any activated sludge plant optimization will be especially sensitive to the choice of a suitable final clarifier model.

Middleton and Lawrence [1975] investigated the sensitivity of a least cost activated sludge system design. The final settling tank was incorporated into the design of the activated sludge subsystem. The sludge handling facility (i.e. gravity thickening, anaerobic digestion, vacuum filtration, and ultimate sludge disposal) was optimized separately. The assumptions of perfect final clarification and constant removal efficiency in the primary settler restrict their analysis but allow efficent optimization of the combined subsystems since sludge age is the only decision variable common to both the liquid and solids handling portions.

All designs were very insensitive to variations in the sludge age, prompting the authors to conclude that sludge age and, therefore, process stability could be increased without trading off economic efficiency. The assumption of perfect final clarification probably contributed to the insensitivity of the solution to sludge age. The recycle ratio and hydraulic retention time in the aeration tank were shown to increase greatly with an increase in influent waste strength. The MLSS concentration and recycle ratio were sensitive to changes in the sludge thickening characteristics, while values of sludge treatment variables were insensitive to these changes. In reality, sludge treatment variables would be expected to change with a change in activated sludge thickening characteristics because the concentration of the waste activated sludge directly affects the input state of the solids handling subsystem. System cost was also insensitive to these parameter variations. The liquid treatment train was not affected by variations in sludge disposal costs, and solids handling costs varied as expected in response to changes in this parameter. Sensitivity coeficients were not calculated from the results of the analysis.

Voelkel [1978] mathematically modeled a wastewater treatment plant consisting of: primary clarification, activated sludge, final clarification, gravity thickening of primary sludge, air floatation thickening of waste activated sludge, anaerobic digestion, chemical conditioning, vacuum filtration, and land disposal of dewatered sludge. Removal of suspended solids in the final clarifier was modeled using the equation developed by Tarrer et al. One unique aspect of Voelkel's research is the inclusion of recycle streams from the solids handling unit processes to the head of the plant. The inclusion of recycle streams is a significant advance over previous wastewater treatment plant optimization models because, in practice, the recycle streams may yield a significant fraction of the suspended solids load on an activated sludge plant [Lawler and Singer, 1984].

However, the recycle streams constitute an internal loop, and thus optimization of the entire system may be difficult because a straightforward solution of the equations is not possible. In fact, Voelkel did experience difficulty in attempting to optimize the system of 88 equations and 97 variables with a version of the Golden Complex Search.

A perturbation analysis was performed on 63 technological parameters and on influent conditions. However, Voelkel's analysis was hindered because the optimization strategy often could not find a feasible point for perturbed values of the parameters. Voelkel also experienced problems with the algorithm when optimizing at nominal values of the parameters (i.e. base conditions). Thus, it is not clear whether the perturbed values for which a feasible point could not be found actually create (mathematically) a null feasible space or whether this indicates a lack of robustness in the optimization procedure.

Normalized sensitivity coefficients were calculated when the optimization results were judged reasonable. All partial derivatives presented have a positive sign, but it seems that some of them should be negative. The same starting point was apparently used for every perturbation and no discussion of local optima was included.

Voelkel ranked the perturbed parameters by the magnitudes of their calculated normalized sensitivity coefficients, if possible. Other criteria were the number of nonfeasible points encountered during the perturbation analysis and the degree of nonlinearity of the response contour.

The method of ranking the parameters by the number of infeasible optimization runs apparently is based on conjecture. Although a parameter for which the Golden Complex Search algorithm was unable to generate a feasible solution may be critical to the optimal design, this cannot be known with certainty.

Chang [1967] derives methods for directly calculating sensitivity coeficients using the definition of sensitivity as a rate of change. Chang derives sensitivity equations for systems described by sets of algebraic, differential, and difference equations, and also for systems optimized via Pontryagin's maximum principal. Two different types of sensitivity analysis are presented, designated type A and type B. The type A analysis requires the optimal decison vector to be fixed, and the type B analysis requires the decision vector to change optimally with a small change in a parameter value. The latter method is more useful since valuable information concerning the sensitivity of the values of the decision variables to a variation in a parameter value is obtained. The approach in both cases involves solving a set of simultaneous linear algebraic equations to obtain the sensitivity coefficients of all state and decision variables (only state variables for the first analysis described). The objective function sensitivity coefficient is then directly calculable. Chang applies this sensitivity analysis to a simple continuous and stagewise process to illustrate its use.

Chang also investigates ways of including sensitivity information into the optimization process, by means of a modified objective function and a method of including sensitivity constraints in the model formulation. Both methods strive to achieve a design which is nearly optimal, but less sensitive than the least cost design to variations in the parameter values. Chang applies these methods to the optimal design of a chemical reactor with one degree of freedom. The least cost design and the system cost were very sensitive to variations in the heat transfer coefficient. When either sensitivity constraints or an augmented objective function containing sensitivity information was applied to the system design, it was possible to reduce the sensitivity of the design considerably with a comparatively small increase in the total cost.

Chang also investigated means of optimizing processes which are subject to uncertainty in the parameter values. If a design is sensitive to a parameter value which has a high degree of uncertainty, the problem becomes one of how to overdesign the process optimally. An optimization procedure is presented which minimizes the expected value of the objective function given the probability density function of the uncertain parameter. This method is applied to a simple reactor design described by first order kinetics where the kinetic rate coefficient is uncertain.

Chen et al. [1970] present the results of a sensitivity analysis of an activated sludge system model in which the aeration basin was modeled as a series of n completely-mixed tanks. They present four ways to incorporate parameter uncertainty into the optimization process. Employed in their analysis is an approach that minimizes the expected value of the objective function. Only the activated sludge subsystem was modeled; primary sedimentation and sludge handling unit processes were neglected. Perfect clarification of active sludge was assumed in the final settler, and the influent waste was assumed to consist only of soluble material. The concentration of settled activated sludge in the recycle stream was calculated by multiplying the biomass concentration in the influent to the final clarifier by a factor, β, the settler concentration efficiency. The objective function was to minimize the total aeration basin volume.

A sensitivity analysis of the second type derived by Chang (type B), in which the decision variables can vary, was used to calculate sensitivity coefficients of the optimal decision and state variables, for variations in several model parameter values. Chen et al. derive an expression for the sensitivity of the optimal objective function value to a variation in a single parameter value, for the type B case. Their result is identical to Chang's expression for the objective sensitivity when the optimal decision variables are fixed (type A). This interesting result indicates that the objective function sensitivity coefficients for both the type A and type B cases are identical.

The optimal total aeration tank volume was very sensitive to variations in the settler concentration efficiency, β, and the recycle ratio, and very insensitive to the dimensionless cell yield (defined as the cell yield divided by the expected value or nominal cell yield), and the endogenous decay rate. Using the propagation of variance concept Chen et al. optimized the system under parameter uncertainty. They considered two different distributions to represent the degree of uncertainty in each parameter: normal and uniform. The expected optimal holding time based on the uniform distribution was greater than that based on the normal distribution, correctly reflecting the larger uncertainty in parameters given by the larger variance of the uniform distribution.

Although the work by Chen et al. involves many unrealistic assumptions which restrict the practical application of their results, it clearly illustrates a method of performing a sensitivity analysis and using these results to incorporate parameter uncertainty into the optimal design. This method can be used to develop quantitative safety factors for design variables (also see Berthouex and Polkowski [1970]) from the ratios of the optimal values of the design variables when parameter uncertainty is considered to their optimal values when parameter uncertainty is ignored.

Tyteca [1981], and Tyteca and Smeers [1981] investigated the optimal design of a secondary wastewater treatment plant consisting of: primary sedimentation, activated sludge, gravity thickening, anaerobic digestion, and vacuum filteration. Their model contained 43 equations and 51 variables and was optimized by a generalized reduced gradient algorithm. In many respects, Tyteca and Smeers' model is similar to the model developed by Tang [1984], which is the basis for the results presented in Chapter 6. There are some differences between the two models, however. One of the most significant differences is that Tyteca and Smeers assume perfect clarification of the waste activated sludge, but Tang uses a final clarifier model to predict solids removal efficiency.

Tyteca [1981] performed a sensitivity analysis using the wastewater treatment plant model. He investigated changes in influent wastewater characteristics, effluent quality, discount rate, and methane recovery in the anaerobic digester, and noted how the optimal design was affected. A perturbation analysis was used to obtain the sensitivity information.

Tyteca found the primary settler area to be very sensitive to changes in the influent or effluent conditions. When the influent suspended solids concentration was low, the primary clarifier was no longer economically justified. The plant design and cost were fairly insensitive to changing effluent BOD_5 requirements, except when the effluent requirement was very strict. The design was also fairly insensitive to the discount rate, although some expenses involving energy useage were affected. In general, parameter variations had little effect on the design of the sludge handling facilities. One exception was methane recovery in the digester. Although the optimal design was quite sensitive to this parameter, total cost remained about the same.

CHAPTER 3

THE WASTEWATER TREATMENT PLANT MODEL

3.1. Model Description

The mathematical wastewater treatment plant model used in this research is described elsewhere in detail [Tang, 1984]. A brief overview of the process model equations and cost information is presented below.

The treatment train modelled is shown in Figure 3.1. It consists of the following unit processes: primary clarification, activated sludge with final clarification, gravity thickening of mixed primary and waste activated sludge, primary and secondary anaerobic digestion, vacuum filtration, and final sludge disposal via sanitary landfill. The model has nine degrees of freedom. Sixty-four state and decision variables and 55 equality and 3 inequality constraints are used to describe the unit processes, recycle streams from sludge handling facilities, effluent requirements, and the minimum air flow rate for mixing in the aeration basin. The plant influent flow and contaminant concentrations are assumed to be steady-state.

Unit process sizes and flow, soluble BOD_5 concentration, and solids concentrations at each control point (Figure 3.1) describe the state of the system for a given decision vector. Thus:

Q_j is the flowrate at control point j, m^3/hr,

S_j is the soluble BOD_5 concentration, g/m^3,

M_{a_j} is the active biomass concentration,

M_{d_j} is the biodegradable volatile solids concentration,

M_{i_j} is the inert volatile solids concentration,

M_{f_j} is the fixed solids concentration,

M_{t_j} is the total solids concentration,

j is the index of the control point, $j=1,2,...,16$.

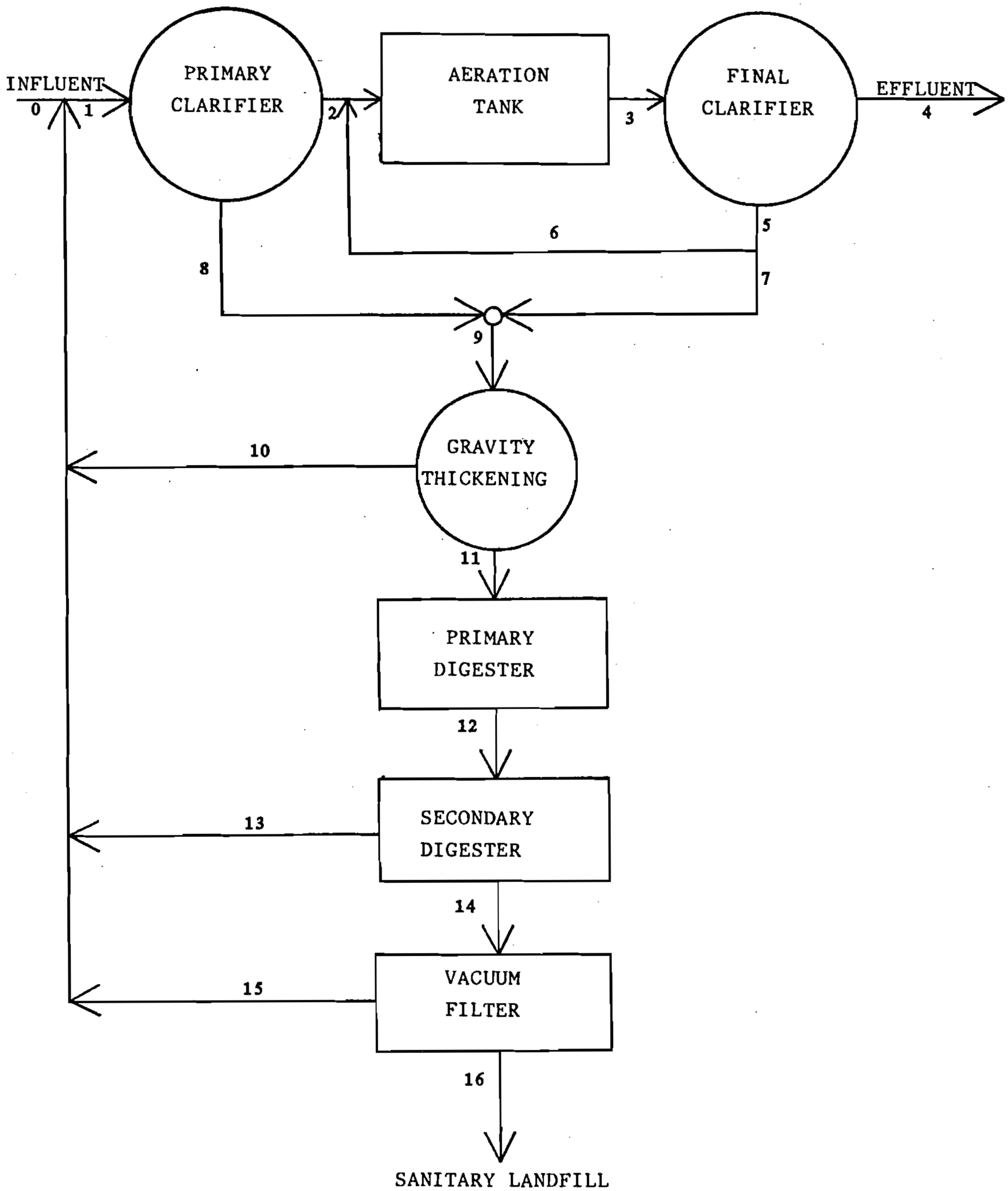

Figure 3.1 - Wastewater treatment plant flow diagram

All solids concentrations are in kg/m^3 unless otherwise specified.

Nine model variables were designated by Tang as decision variables; these are presented in Table 3.1 with the bounds imposed upon them during the optimization. These bounds reflect limits on process model validity or practical considerations. The lower bound of zero on the recycle ratio, RR, represents a relaxation of the 10% lower bound used in Tang's research. A listing of all model variables and associated nomenclature is in Appendix A.

Table 3.1. - Decision Variables and Bounds

Variables	Lower Bound	Upper Bound
Overflow Rate, Primary Clarifier (OR_p), m/hr	0.5	6.0
Sludge Age, Aeration Tank (θ_c), days	2.0	6.0
Hydraulic Retention Time (θ_{at}), days	0.1	0.5
Sludge Recycle Ratio (RR)	0.0	1.0
Solids Loading, Gravity Thickener (L_{gt}), $kg/day/m^2$	12.0	48.0
Digester Temperature (T_d), °C	20.0	60.0
Sludge Age, Primary Digester (θ_d), days	5.0	30.0
Solids Loading, Secondary Digester (L_d), $kg/day/m^2$	12.0	48.0
Vacuum Filter Yield (L_f), $kg/m^2/hr$	5.0	50.0

Parameters in the model are separated into two catagories: technological parameters and cost coefficients. Technological parameters include such things as kinetic coefficients and plant influent conditions. Cost coefficients include the coefficient and exponent of each unit process cost function. All technological parameters are listed in Table 3.2 with nominal values that describe the base plant conditions.

3.2. Process Models

Solids removal in the primary settling tank is assumed to follow the model of Voshel and Sak [1968]:

$$\frac{(M_{t_1} - M_{t_2})}{M_{t_1}} = 0.139(M_{t_1})^{.27}\left(\frac{Q_2}{A_p}\right)^{-0.22} \tag{3.1}$$

Table 3.2 - Nominal values of model technological parameters

Parameter (Units)	Nominal Value
Economic Data	
Capital Recovery Factor	0.09716
Base (1971) Cost Index	150.6
Cost Index for 1980	362.0
Operating/Maintenance Wages (dollars/hr)	8.9
Land Cost, C_L (dollars/acre)	5000.00
Electricity Cost (dollars/kWhr)	0.05
Pumping Head, H (meters)	10.0
Pumping Efficiency, ϵ_p	0.6
Primary Sedimentation	
Constant in Voshel-Sak Model, v_1	0.139
Constant in Voshel-Sak Model, v_2	0.27
Constant in Voshel-Sak Model, v_3	0.22
Sludge Settling Characteristics	
Thickening Constant, a_w	24.24
Thickening Constant, a_p	198.68
Thickening Constant, a_2	2.5
Thickening Constant, n_w	2.3747
Thickening Constant, n_p	2.803
Activated Sludge Kinetics	
Growth Yield Coefficient, Y (g cells/g BOD_5)	0.4
Half-Velocity Constant, K_s (g BOD_5/m^3)	60.0
Maximum Specific Utilization Coefficient, k (day^{-1})	5.0
Endogeneous Decay Coefficient, b (day^{-1})	0.04
Fraction of Cells Degradable, f_d	0.77
Conversion (g BOD_L/g cell)	1.42
Conversion (g BOD_L/g BOD_5)	1.5
Secondary Sedimentation	
Constant in Chapman Model, c_1	5.69
Constant in Chapman Model, c_2	0.00403
Constant in Chapman Model, c_3	11.91
Aeration	
Alpha Factor in Aeration	0.8

Parameter (Units)	Nominal Value
Beta Factor in Aeration	0.95
D.O. Concentration in Aeration Tank, DO_{at} (g/m^3)	1.5
D.O. Saturation Concentration, C_s (g/m^3)	9.17
Temperature of Mixed Liquor, T_L ($^\circ C$)	20.0
Oxygen Transfer Efficiency, OTE	0.08
Density of Air, ρ_{air} (kg/m^3)	1.2
Weight Fraction of Oxygen in Air, γ	0.232
Mixing Requirement, η (m^{3}air/m^3/min)	0.02
Gravity Thickening	
TSS of Thickener Supernatant, $M_{t_{10}}$ (kg/m^3)	0.2
Anaerobic Digestion	
Coefficient in Reaction Rate Expression, R_1	-3.3333
Coefficient in Reaction Rate Expression, R_2	9.8105
Temperature of Digester Influent, T_o ($^\circ C$)	20.0
Methane Production (m^3/kg BOD_L)	0.35
Average Ambient Temperature, T_a ($^\circ C$)	10.0
Efficiency of Heat Exchanger, ϵ	0.85
Heat Conduction Coefficient, U (W/m^2– $^\circ C$)	1.0
Outside Surface Area and Volume Ratio for Digester, a	0.4
Worth of Digester Gas ($/10^{6}kJ)	2.5
Soluble BOD_5 in Digester Supernatant, S_{12} (g/m^3)	500.0
Factor Accounting for the Effect of Rising Gas on Thickening in Secondary Digester, δ	0.25
Thickening Constant for Digested Sludge, a_d	292.6
Thickening Constant for Digested Sludge, n_d	2.9
TSS of Digester Supernatant, $M_{t_{13}}$	4.0
Height of Digester (meters)	10.0
Vacuum Filtration	
Form Time per Cycle Time, χ	0.33
Pressure Applied on Vacuum Filter, P (Nt/m^2)	83300.0
Viscosity of Filtrate, μ (Nt-sec/m^2)	8.9×10^{-4}
Cycle Time, t_c (min)	6.0
Specific Resistance of Sludge, r_s (m/kg)	1.0×10^{12}
TSS of Filtrate, $M_{t_{15}}$ (kg/m^3)	2.0
Effluent Standards	
BOD_5 Concentration (mg/l)	30.0
TSS Concentration (mg/l)	30.0

Parameter (Units)	Nominal Value
Plant Influent Conditions	
Wastewater Flow, Q_o (m^3/hr)	1500
Soluble BOD_5, S_o (mg/l)	100
Active Biomass, M_{a_o} (mg/l)	5
Degradable Suspended Solids, M_{d_o} (mg/l)	100
Inert Suspended Solids, M_{i_o} (mg/l)	45
Fixed Suspended Solids, M_{f_o} (mg/l)	50

A_p = area of the primary clarifier.

Thus the suspended solids removal efficiency is a function of both influent solids concentration and overflow rate. This relationship was developed from plant scale studies. In general, theoretical relationships were preferred when available, but the settling theory of non spherical particles in plant scale basins is not fully developed at this time.

The thickening of sludge in the clarifiers and gravity thickener is predicted by the model of Dick and Suidan [1975], which was derived from limiting flux theory using the expression for batch settling velocity of Duncan and Kawata [1968]. The thickening model is:

$$M_{t_u} = [a(n-1)]^{\frac{1}{n}} \left(\frac{n}{(n-1)}\right) \left(\frac{Q_u}{A_t}\right)^{\frac{-1}{n}} \tag{3.2}$$

M_{t_u} is the underflow solids concentration;

a is a thickening constant,

n is a thickening constant,

Q_u is the underflow flow rate from the thickening unit,

A_t is the area of the thickener or clarifier.

Removal of soluble BOD_5 in the aeration tank is assumed to follow Modod kinetics, and the equations developed by Lawrence and McCarty [1970] are used in the activated sludge system design. These equations are well known and consequently not cited here. Dissolved oxygen requirements in the aeration tank are predicted using the model developed by Lawrence and McCarty.

The final clarifier performs two functions vital to the performance of an activated sludge plant: clarification and thickening of the waste activated sludge. Because of the interdependence of the aeration tank performance and final settler configuration, they are designed integrally as one unit process. Also, since effluent suspended solids contribute heavily to effluent BOD, a model predicting solids removal efficiency must be included. The Chapman model [1983] is used to predict clarification of activated sludge in the final settler:

$$M_{t_4} = -180.6 + (4.03\text{x}10^{-3})M_{t_3} + 133.24\left(\frac{Q_3}{A_f}\right)$$

$$+ SWD\left[90.16 - 62.54\left(\frac{Q_3}{A_f}\right)\right] \tag{3.3}$$

A_f is the area of the final clarifier,
SWD is the side water depth of the clarifier,
M_{t_4} is in g/m^3.

Tang assumes the *SWD* to be constant and thus the final clarifier efficiency is a function of influent suspended solids (MLSS) concentration and overflow rate.

Primary and waste activated sludge are mixed and then thickened in a gravity thickener prior to anaerobic digestion. The underflow solids concentration is predicted from equation 3.2 and the thickening characteristics of the combined sludge, a_c and n_c, are a function of the fraction of primary sludge in the mixture. Solids concentration in the supernatant is assumed constant.

The primary anaerobic digester is modeled as a chemostat and the first order kinetic stabilization rate is a function of the digester temperature. An equation was fit to data compiled by Wise [1980] which plots linearly on a semi-logarithmic graph of K_1, the digester rate coeficient, versus the inverse of temperature. The expression describing this relationship is presented with a plot of the data in Figure 3.2. The total volatile solids entering the digester is equal to the total suspended solids concentration from the gravity thickener less the fixed solids concentration, which is not degradable. Since the data plotted in Figure 3.2 are based on total volatile solids concentration, no correction is made to account for the fraction of total volatile solids which are amenable to biological degradation.

The heat requirement in the digester is a function of: sludge flow rate, influent and effluent sludge temperature, temperature of the surrounding air, total external area of the digester, and efficiency of the heat exchanger. Methane gas production is proportional to the volatile solids destruction and is included in the cost function as a credit. The secondary digester is modeled as a

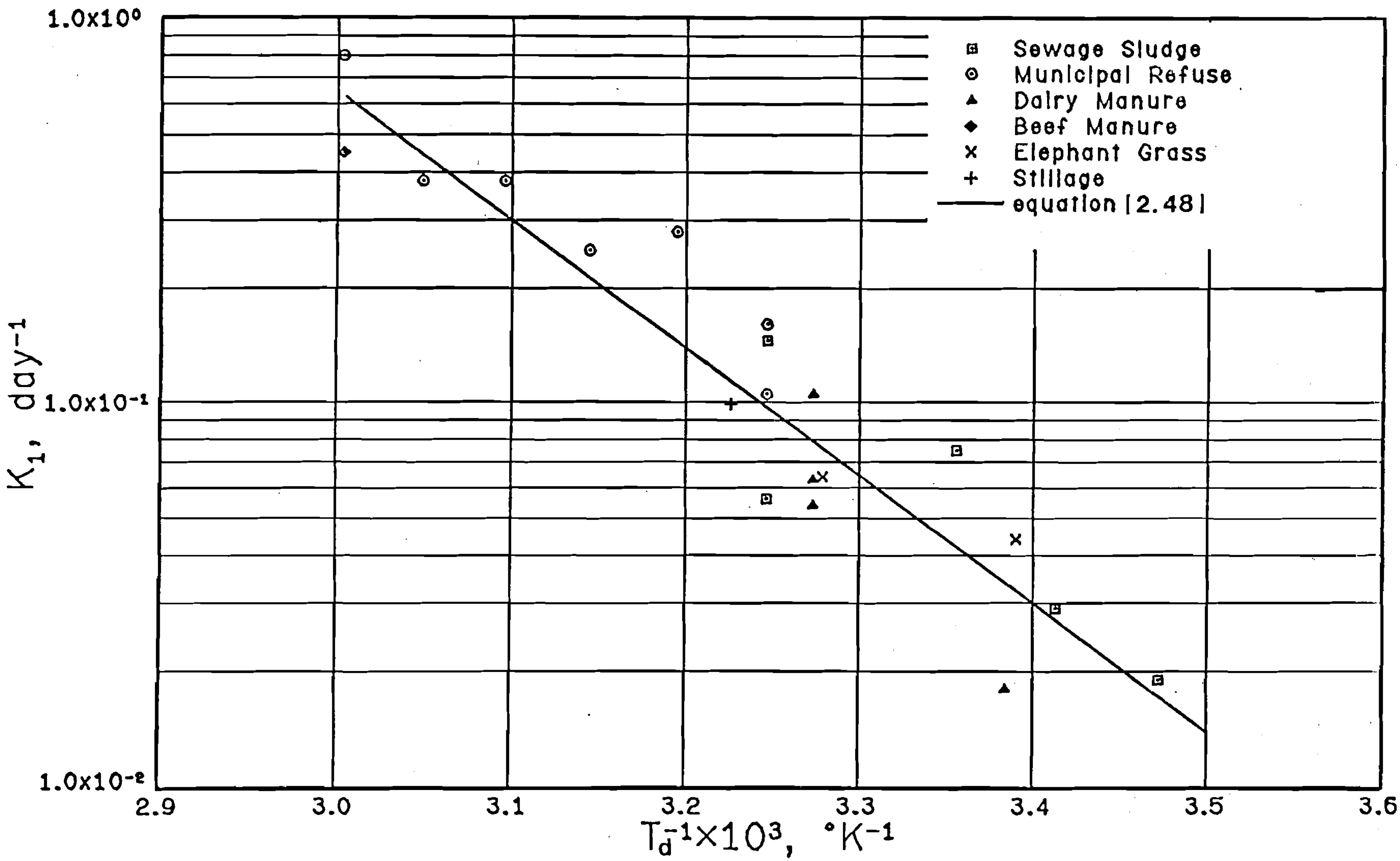

Figure 3.2 - Primary digester first order reaction rate as a function of digester temperature

gravity thickener, except that the thickening rate is assumed to be one fourth that of a fully digested sludge to account for buoyant properties of the incoming sludge due to gas production. Solids concentration in the supernatant is assumed constant.

Sludge dewatering is performed by vacuum filtration and design of the vacuum filter is based on the filtration theory of Coackley and Jones [1956]. Filtrate solids concentration is assumed constant.

3.3. Cost Functions

The objective function to be minimized is the sum of all capital, maintenance, and operating expenses of the wastewater treatment plant. All costs are on an annual basis for an assumed plant design life and interest rate.

The cost of sludge disposal is a function of the landfill area required and this area is estimated by Dick et al. [1978] to be:

$$AL(\text{acres}) = 3.62\text{x}10^{-2}(Q_{16}M_{t_{16}}) \tag{3.4}$$

AL is the area of landfill required in acres.

The cost function is given by Rossman [1980] as:

$$\begin{aligned} \textit{Capital Cost}(1980\ \$) &= (\textit{landcost},\ \$/\textit{acre})AL \\ &\quad + 6200q^{0.74}(\textit{cost update factor}) \end{aligned} \tag{3.5}$$

q is the wet tons of sludge applied per day, which equals $27.5132Q_{16}$ for a sludge with a specific gravity of 1.04.

All other cost functions for the unit processes have the general form:

$$C = a(X)^b \tag{3.6}$$

C is the capital, operation, materials and supply, or power cost,

a is the cost coefficient,

b is the cost exponent,

X is the model variable related to size or capacity of the process.

CHAPTER 4

OPTIMIZATION STRATEGY AND COMPUTATIONAL EXPERIENCES

A Generalized Reduced Gradient algorithm, GRG2, developed by Lasdon et al [1979] was used to optimize the total system model. GRG2 has been shown to be very robust and therefor useful for solving large, highly nonlinear NLP's.

4.1. Solution Accuracy

There are several ways in which GRG2 may terminate the optimization procedure. The highest degree of confidence is obtained when the Kuhn-Tucker conditions for optimality are met at the final point. This implies the necessary conditions for a local optimum have been met to within a certain user defined tolerance. GRG2 may also terminate when the fractional change in the objective is less than a user specified value for a user specified number of iterations. This termination criterion implies less confidence in the final solution accuracy. For every optimization run presented herein, except where noted, the second termination criterion was met. This result does not necessarily mean the final decision vector is not a good solution. It may, in fact, closely approximate either a locally or globally optimal solution to the problem. The higher confidence termination criterion may not be satisfied because of inadequate scaling of the problem functions and variables [Lasdon et al., 1982] or because of the inherent difficulty in simultaneously solving the set of binding nonlinear constraints at each step. Since the wastewater treatment model consists mainly of equality constraints, this latter difficulty is expected to be significant.

Experience in working with the current model and knowledge of the problem led to developement of informal criteria used to assess the accuracy of the final solutions. It was observed that an optimal solution usually possessed the following seven characteristics:

[1] Low sludge age (θ_c less than 3.0 days)

[2] High primary clarifier overflow rate (OR_p often at upper bound of 6.0 $m^3/m^2/hr$)

[3] Low recycle of activated sludge (RR less than 20 percent)

[4] Low solids loading on the gravity thickener (L_{gt} often at lower bound of 0.5 $kg/m^2/hr$)

[5] High temperature of primary anaerobic digester (T_d at upper bound of 60 ℃)

[6] High solids loading on secondary digester (L_d often at upper bound of 2.0 $kg/m^2/hr$)

[7] High concentration of vaccum filter cake ($M_{t_{16}}$ at upper bound of 150.0 kg/m^3)

Thus, if several optimization runs were made, each starting at a different feasible point in the problem domain, and if the results from these optimizations were very similar and possessed the above seven characteristics, then this was symptomatic of a solution which closely approximated a local and possibly the global optimum. However, proving a solution is globally optimal is very difficult for complex, highly nonlinear systems.

4.2. Algorithmic Option-PARSH Subroutine

Because GRG2 uses a gradient method to solve a sequence of reduced problems, partial derivatives of all problem functions with respect to all variables are required. One feature of GRG2 is the option of calculating these derivatives numerically using either forward or central finite differences. This convenience comes at the expense of higher computing costs and decreased accuracy. Higher computing costs result from the extra function calls needed at each iteration to compute the numerical derivatives. With the exception of quadratic functions, either numerical approximation of the derivative will be less accurate than an analytical gradient expression. For these two reasons an optional subroutine was written to calculate the partial derivatives analytically. Derivatives obtained using the analytical expressions were compared to estimates obtained using central finite differences, and one set of duplicate runs was made to assess the computational advantages of using the analytical expressions. On a HARRIS 800-2 virtual memory machine the run using numerical derivatives took 285 seconds of CPU time and the identical run using analytical derivatives took 248 seconds of CPU time. Thus, for this set of comparative runs the analytical expressions yielded a 13 percent reduction in CPU time.

One caution regarding coding of the analytical derivatives is warranted. In the initial development of the subroutine all constant derivatives were coded to be calculated only once, during the first iteration, via the use of a GRG2 supplied iteration variable. This was done to reduce the cost of calculating the derivatives at every iteration. In fact, Lasdon et al. [1978] state "Even greater reductions (in CPU time) could have been obtained by coding the partial derivative subroutine PARSH to evalutate the constraint gradients (which are constant) only once." However, the subroutine implementing this idea consistantly caused the optimization to terminate prematurely at a non-optimal point. It was discovered that, after the first iteration, the values of several constant derivatives had been changed. Presumably, GRG2 uses array storage allocated to the gradient array for other purposes elsewhere in the program. Thus all gradients, constant or otherwise, should be explicitly calculated during each call to the PARSH subroutine.

4.3. Machine Dependent Accuracy

GRG2 was run on two different computers at the University of Illinois: a Control Data Corporation CYBER 170 series mainframe, and the previously mentioned HARRIS minicomputer. GRG2 runs on the CDC machine in single precison with 60 bit data words, and on the HARRIS machine in double precision with 48 bit data words. The solutions obtained on the CYBER were consistantly better (although only slightly) than those obtained from identical runs on the HARRIS. Although the difference could be partially attributed to the increased number of significant digits carried by the CDC machine, identical runs still produced slightly inferior results on the HARRIS even when using 96 bit data words (quadruple precision). The values of several machine-dependent options and tolerances within GRG2 could affect the final solution. It is notable that the accuracy of a particular solution may be affected by the computer used.

GRG2 proved a useful tool for solving this particular medium size, highly nonlinear program. The system model could be optimized, in its entirety, even given the presence of the recycle mass balances and the large number of nonlinear equality constraints to be satisfied. However, before a program will run efficiently and consistently, much work may be required. The straightforward implementation of GRG2 to a particular problem is fairly easy, but the accuracy and efficiency of the final solution may be highly dependent upon the scaling of problem functions and variables, and upon the choices of many tolerances and algorithmic options available to the GRG2 user.

CHAPTER 5

SENSITIVITY THEORY AND METHODS OF ANALYSIS

5.1. Introduction

Usually, in the design or analysis of engineering systems, the values of many parameters are required. These values are often not known with certainty, and must be estimated from experience, or by conducting (sometimes expensive and elaborate) research experiments. Under actual operating conditions, these parameters may take on values which are different than those values estimated from experience or experimentation. In addition, the parameters assumed constant may really have stochastic properties; they may vary in time over a certain range. These shifts in parameter values can combine to alter significantly the performance of the real process.

Associated with every engineered project is an element of risk. The engineer will attempt to minimize the risk, often by using "safety factors," which multiply values of design variables to obtain a final design which is not as economically efficient, but which is less sensitive to parameter variations. Thus, parameter uncertainty is associated with a fundamental tradeoff in engineering design: increased economic efficiency versus decreased risk.

Sensitivity analysis applied to a model can determine to what degree performance of the system depends on certain parameters. If the system outputs are very insensitive to variations in one parameter, the designer need not include an additional safety factor to reduce the risk imposed by uncertainty in that parameter value. However, if the system outputs are very sensitive to variations in a parameter, the designer may wish to reduce risk either by additional research to obtain higher confidence in the value, or by introducing an appropriate safety factor to compensate for the parameter uncertainty.

Sensitivity information is therefor very important to the designer because it can reduce the additional cost burden associated with applying arbitrary safety factors to all design variables. Sensitivity analysis helps the engineer assess the risks involved with a particular system design.

A sensitivity analysis of a properly formulated model will yield valuable information concerning how operating or environmental conditions may be altered to produce a more cost effective design. Sensitivity information also indicates which direction parameter values should be perturbed to produce a design safety factor. The direction of perturbation may be intuitive to the experienced engineer, but possibly not, if the system is complex.

The art of modeling itself is intimately associated with sensitivity. Sensitivity measures the response of a system output to changes in a system input; modeling attempts to mimick such responses in the real world. Thus, sensitivity can tell the modeler to what degree observed reality is mirrored by the process description, and where deficiencies in the model may lie. Sensitivity analysis can and should be used as an integral part in every stage of the modeling process.

Mathematical models are particularly well suited to sensitivity analysis, as the sensitivity information can be obtained relatively easily and inexpensively via computer simulation, optimization algorithms, or sensitivity equations such as those presented in this chapter. Both perturbation and sensitivity equation methods are described below, and their relative advantages and limitations discussed.

5.2. Definition of Sensitivity

Sensitivity is a rate of change in one factor with respect to a rate of change in another factor. This general definition implies the quotient of two differentials. The sensitivity of system outputs to changes in model parameter values is one specific application of the sensitivity concept. Another application is in model building and verification. For example, the mathematical form of a model determines the sensitivity of the model output to changes in the values of model variables, and this sensitivity should agree with observed data. Derivation of the mathematical foundations of sensitivity is applicable to all phases of the modeling process.

5.3. Mathematical Basis for Sensitivity

The mathematical definition of sensitivity is derived from consideration of the Taylor series expansion of the explicit function:

$$F_0 = X(F_1, F_2, \ldots, F_n) \tag{5.1}$$

The change in factor F_0 resulting from a change in factor F_i can be expressed:

$$X(F_i + \Delta F_i, F_j \big|_{j \neq i}) = F_0 + \Delta F_i \frac{\partial F_0}{\partial F_i} + \frac{1}{2!} \frac{\partial^2 F_0}{\partial F_i^2} \Delta F_i^2 + \cdots \tag{5.2}$$

If second order and higher terms are considered negligible then equation 5.2 reduces to:

$$X(F_i + \Delta F_i, F_j \big|_{j \neq i}) = F_0 + \Delta F_i \frac{\partial F_0}{\partial F_i} \tag{5.3}$$

Thus:

$$X(F_i + \Delta F_i, F_j \big|_{j \neq i}) - F_0 = \Delta F_i \frac{\partial F_0}{\partial F_i} \tag{5.4}$$

Equation 5.4 is referred to as the linearized sensitivity equation [McCuen, 1976], and it approximates the change resulting in factor F_0 due to a corresponding change in factor F_i. The general definition of sensitivity is derived from equations 5.1 and 5.4:

$$S = \frac{[X(F_i + \Delta F_i, F_j \big|_{j \neq i}) - X(F_1, F_2, \ldots, F_n)]}{\Delta F_i} = \frac{\partial F_0}{\partial F_i} \tag{5.5}$$

Where S is the sensitivity coefficient, and denotes the rate of change of factor F_0 to factor F_i.

Equation 5.5 implies two types of sensitivity analysis; the first term suggests the method of factor perturbation, and the second term suggests a direct differentiation method. The method of factor perturbation is by far more common since evaluating the differentials can often be an appreciable chore for large complex systems.

The senstivity coefficient in equation 5.5 can be modified by dividing the numerator by F_0 and the denominator by F_i:

$$S_n = \frac{\partial F_0 / F_0}{\partial F_i / F_i} = \frac{\partial F_0}{\partial F_i} \frac{F_i}{F_0} \tag{5.6}$$

Equation 5.6 defines the normalized or relative sensitivity coefficient [McCuen, 1976], S_n, and is the coefficient calculated in this work. The normalized sensitivity coefficient is useful for comparative purposes, because it gives an estimate of the relative change in factor F_0 due to a relative change in factor F_i. The interpretation is, for $S_n = 1.0$, a change in factor F_i of one percent will affect a change in factor F_0 of one percent. The direction of this change is given by the sign of the sensitivity coeficient.

5.4. Sensitivity Equations

Chang [1967] has derived sensitivity equations for two cases: 1) when the optimal decision vector is fixed, and 2) when the decision variables are allowed to vary in response to a parameter variation. These sensitivity equations are presented below. A system described by algebraic equations can always be written in the form:

$$\textit{Minimize } J = f(\mathbf{x}, \boldsymbol{\theta}, \boldsymbol{\xi}) \tag{5.7}$$

$$\textit{Subject to: } g_k(\mathbf{x}, \boldsymbol{\theta}, \boldsymbol{\xi}) = 0 \qquad j = 1, 2, \ldots, s$$

J is the objective function to be minimized,

g_k is the k^{th} constraint describing the system,

$\mathbf{x}$ is a s-dimensional vector of state (dependent) variables,

$\boldsymbol{\theta}$ is a r-dimensional vector of decision (independent) variables,

$\boldsymbol{\xi}$ is a p-dimensional vector of model parameters.

5.4.1. Type A-Decision Vector Fixed

The purpose of this type of analysis is to find $\frac{\partial J}{\partial \xi_i}$, the sensitivity of the objective to a small change in parameter i, when the decision vector is fixed. Differentiating J with respect to ξ_i gives:

$$\frac{\partial J}{\partial \xi_i} = \sum_{j=1}^{s} \frac{\partial f}{\partial x_j} \frac{\partial x_j}{\partial \xi_i} + \frac{\partial f}{\partial \xi_i} \tag{5.8}$$

The unknowns in the above equation are the sensitivities of the state varibles with respect to ξ_i, *i.e.* $\frac{\partial x_j}{\partial \xi_i}$ $j = 1,2,\ldots,s$. These sensitivities are obtained by differentiating the constraints with respect to ξ_i, as follows:

$$\sum_{j=1}^{s} \frac{\partial g_k}{\partial x_j} \frac{\partial x_j}{\partial \xi_i} + \frac{\partial g_k}{\partial \xi_i} = 0 \qquad k = 1,2,\ldots,s \tag{5.9}$$

The above sensitivity equation represents a system of linear algebraic equations which can be solved for $\frac{\partial x_j}{\partial \xi_i}$ $j = 1,2,\ldots,s$. From equations 5.8 and 5.9, in vector matrix notation the objective function sensitivity is:

$$\frac{\partial J}{\partial \xi_i} = -\left\{\frac{\partial f}{\partial \mathbf{x}}\right\}^T \left[\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]^{-1} \left\{\frac{\partial \mathbf{g}}{\partial \xi_i}\right\} + \frac{\partial f}{\partial \xi_i} \tag{5.10}$$

In the above equation { } denotes a column vector, [ ] denotes a square matrix, T denotes the transpose, and $[\ ]^{-1}$ denotes the inverse of a matrix. All derivatives are evaluated at a set of decisions, $\boldsymbol{\theta}$, and at the nominal parameter values. The objective sensitivities may be normalized for comparative purposes (see equation 5.6).

5.4.2. Type B-Decision Vector Free

The purpose of this type of analysis is to determine the sensitivity of J when the optimal decision vector is not fixed. In addition, valuable information concerning the sensitivities of the decision variables to changes in model parameters is obtained.

Chen et al. [1970] present a derivation of the objective sensitivity for the Type B sensitivity analysis. The resulting expression is identical to equation 5.10; Chen et al. indicates that $\frac{\partial J}{\partial \xi_i}$ is the same regardless of whether the decision vector can change. This result is counterintuitive for, if the decision vector is fixed, the ability to optimize is lost. If the decision vector is free, a parameter change could induce a change in the optimal design, which would result in a lower (or equal, if alternate optima exist) objective value than for the fixed case. This must be true since the objective is minimized. A practical example illustrates this important concept.

Consider a typical wastewater treatment plant in operation. If suddenly a slug of hard-to-degrade wastewater enters the plant, simulating a parameter change, the cost of meeting the effluent requirements would increase because of operational changes associated with the aeration basin and other unit processes. If the operator could change unit process sizes at will, the annual amortized cost of meeting the effluent requirements could decrease. The original solution is still available to the operator, but new solutions (essentially new plants) are available as well.

The above discussion suggests a different approach to the computation of the objective sensitivity coefficient. Differentiation of J with respect to ξ_i gives:

$$\frac{\partial J}{\partial \xi_i} = \sum_{j=1}^{s} \frac{\partial f}{\partial x_j} \frac{\partial x_j}{\partial \xi_i} + \sum_{l=1}^{r} \frac{\partial f}{\partial \theta_l} \frac{\partial \theta_l}{\partial \xi_i} + \frac{\partial f}{\partial \xi_i} \tag{5.11}$$

The above expression is used in this work to calculate objective sensitivity coefficients for the Type B analysis. A simple example problem has been formulated which indicates this equation is superior, for one specific case, to Chen's derivation. However, research is needed to generalize this result.

The unknowns in equation 5.11 are the sensitivities of the state and decision variables with respect to a change in ξ_i: $\frac{\partial x_j}{\partial \xi_i}$ $j = 1,2, \ldots, s$ and $\frac{\partial \theta_l}{\partial \xi_i}$ $l = 1,2, \ldots, r$. Calculation of these derivatives must be constrained by the mathematical optimality conditions, so that the decision vector is constrained to change optimally with a parameter variation.

For the problem in 5.7, the first order necessary conditions for optimality are:

$$g_k(\mathbf{x},\boldsymbol{\theta},\boldsymbol{\xi}) = 0 \qquad k = 1,2,\ldots,s \tag{5.12}$$

$$\frac{\partial f}{\partial x_j} - \sum_{k=1}^{s} \lambda_k \frac{\partial g_k}{\partial x_j} = 0 = h_j(\mathbf{x},\theta,\lambda,\boldsymbol{\xi}) \qquad j = 1,2,\ldots,s \tag{5.13}$$

$$\frac{\partial f}{\partial \theta_l} - \sum_{k=1}^{s} \lambda_k \frac{\partial g_k}{\partial \theta_l} = 0 = p_l(\mathbf{x},\theta,\lambda,\boldsymbol{\xi}) \qquad l = 1,2,\ldots,r \tag{5.14}$$

Differentiating 5.12 through 5.14 with respect to ξ_i gives:

$$\sum_{j=1}^{s} \frac{\partial g_k}{\partial x_j}\frac{\partial x_j}{\partial \xi_i} + \sum_{l=1}^{r} \frac{\partial g_k}{\partial \theta_l}\frac{\partial \theta_l}{\partial \xi_i} + \frac{\partial g_k}{\partial \xi_i} = 0 \qquad k = 1,2,\ldots,s \tag{5.15}$$

$$\sum_{j=1}^{s} \frac{\partial h_k}{\partial x_j}\frac{\partial x_j}{\partial \xi_i} + \sum_{l=1}^{r} \frac{\partial h_k}{\partial \theta_l}\frac{\partial \theta_l}{\partial \xi_i} + \sum_{j=1}^{s} \frac{\partial h_k}{\partial \lambda_j}\frac{\partial \lambda_j}{\partial \xi_i} + \frac{\partial h_k}{\partial \xi_i} = 0$$

$$k = 1,2,\ldots,s \tag{5.16}$$

$$\sum_{j=1}^{s} \frac{\partial p_k}{\partial x_j}\frac{\partial x_j}{\partial \xi_i} + \sum_{l=1}^{r} \frac{\partial p_k}{\partial \theta_l}\frac{\partial \theta_l}{\partial \xi_i} + \sum_{j=1}^{s} \frac{\partial p_k}{\partial \lambda_j}\frac{\partial \lambda_j}{\partial \xi_i} + \frac{\partial p_k}{\partial \xi_i} = 0$$

$$k = 1,2,\ldots,r \tag{5.17}$$

Equations 5.15 through 5.17 constitute a set of linear, algebraic equations to be solved for the unknowns: $\frac{\partial x_j}{\partial \xi_i}$ $j = 1,2,\ldots,s$, $\frac{\partial \theta_l}{\partial \xi_i}$ $l = 1,2,\ldots,r$, and $\frac{\partial \lambda_j}{\partial \xi_i}$ $j = 1,2,\ldots,s$. The objective sensitivity can then be calculated from equation 5.11. All derivatives are calculated at the optimal decision vector and nominal values of parameters. The sensitivity coefficients may be normalized for comparative purposes (see equation 5.6).

5.4.3. Generalization of Objective Sensitivity Expression

The objective function often is the sum of contributions from individual system components, as:

$$J = \sum_{k=1}^{m} f_k(\mathbf{x},\boldsymbol{\theta},\boldsymbol{\xi}) \tag{5.18}$$

A straightforward extension of equation 5.11 yields the sensitivity expression for this new objective:

$$\frac{\partial J}{\partial \xi_i} = \sum_{k=1}^{m} \sum_{j=1}^{s} \frac{\partial f_k}{\partial x_j} \frac{\partial x_j}{\partial \xi_i} + \sum_{k=1}^{m} \sum_{l=1}^{r} \frac{\partial f_k}{\partial \theta_l} \frac{\partial \theta_l}{\partial \xi_i} + \sum_{k=1}^{m} \frac{\partial f_k}{\partial \xi_i} \tag{5.19}$$

If the objective is segmented in a meaningful way, additional information may be obtained by examining the sensitivity contribution of each k^{th} component individually. This was done in this work to show variation in sensitivity between capital, operating, maintenance, materials and supply, and power costs, and within these groups the variation with respect to each individual unit process.

5.5. Numerical Methods

The optimal solution may be found by any applicable optimization strategy. However, if the vector of lagrange multipliers is not provided by the optimization algorithm selected, it must be determined from simultaneous solution of any s of the $s+r$ necessary conditions given by 5.13 and 5.14. The s necessary conditions must be selected so that the resulting system of equations is non-singular.

Many stratagies could be used to compute the elements of the coefficient matrix of equations 5.15, 5.16, and 5.17. In this research, analytical expressions were written to compute gradients of the objective function and constraints with respect to each variable. The partial derivatives of h_j and p_l were then computed by central finite difference.

The coefficient matrix for the system of linear equations to be solved may be large and very sparse, depending on the model structure. The coefficient matrix for the wastewater treatment plant model considered in this work contained less than 5.0% nonzero elements. In this case, decomposition of the coefficient matrix by standard gaussion elimination techniques may not proceed well. Also, a poorly scaled coefficient matrix will result if the NLP is poorly scaled, and this could affect the solution accuracy.

In this work the Y12M sparse matrix solution package available at the University of Illinois was used to solve the linear system and obtain the sensitivity coefficients. Prior to decomposition, the coefficient matrix and right hand sides were scaled using a sparse matrix scaling routine from the HARWELL software library (subroutine MC19A) available at the University of Illinois.

5.6. Comparison of Sensitivity Equations and Perturbation Analysis Approaches

5.6.1. Accuracy

Both perturbation analysis and the sensitivity equation approach yield information subject to ill-conditioning errors. An NLP exhibits ill-conditioning when optimization proceeds poorly and frequently terminates at a non-optimal point. This behavior may be caused by algorithmic or numerical problems, high interaction between model variables, or poor scaling of model variables and functions. It is common for large, highly nonlinear problems to exhibit some degree of ill-conditioning.

If ill-conditioning prevents obtaining a mathematically optimal solution, the sensitivity analysis might be grossly in error. The degree of error is probably difficult to determine and depends on how closely the final solution approaches a stationary point.

A related source of inaccuracy is due to non-convexity of the objective function or the feasible region. If the necessary optimality conditions have been met but the point is not a globally optimal solution, the sensitivity analysis will not, in general, predict the sensitivity of the global optima to changes in model parameters.

5.6.2. Computational Requirements

For the problem studied, the sensitivity equation approach yielded much information for very little computational expense. The average time required to compute sensitivity coefficients for all model variables and for changes in all technological parameters and cost coefficients was 120 CPU seconds on the HARRIS minicomputer. In comparison, just one optimization run on the same machine required from 250 to 850 CPU seconds. Thus, if the problem is expected to have many local optima or exhibits ill-conditioned behavior, a perturbation analysis might be quite expensive in comparison to the sensitivity equation approach. Several computer runs might be required to reach a desired confidence level or degree of accuracy for each factor perturbation. The sensitivity equations require only that the global optimum be approximated to the desired confidence level. Since effort will be devoted to this end, whether or not post-optimal sensitivity analysis is performed, the sensitivity equation analysis requires very little extra computation.

5.6.3. Information from Sensitivity Equations and Perturbation Analysis

In one sense the two sensitivity methods should not be compared; the information obtained from each is conceptually different. The sensitivity coefficients calculated from sensitivity equations are analagous to derivatives; a first-order approximation of change in the optimal objective function and model variables in response to a parameter variation. The sensitivity coefficients cannot, in general, accurately predict the effect of a large scale change in a parameter or the effect of varying several parameters simultaneously. The first order approximation is sufficient for many real systems [Berthouex and Polkowski, 1970], however, and the normalized sensitivity coefficients allow quick comparison of a large number of parameter effects.

In contrast, Perturbation techniques can be used to examine large-scale changes in parameters and the effect of several parameters varying simultaneously. Perturbation methods do not, however, yield information about the rate of change as accurately as the sensitivity equations.

The two sensitivity methods should be thought of as complements. They both could be used to investigate fully the sensitivity of the model. One possible approach would be to use sensitivity equations during the model building phase to aid in selecting adequate process models. When the model is completed, sensitivity coefficients for a few selected optimal solutions (corresponding to different inputs or outputs) should give good indication of parameters which might be critical to the design. Perturbation methods could then be used to explore more fully the range of tradeoffs associated with certain parameter variations.

CHAPTER 6

SENSITIVITY ANALYSIS OF WASTEWATER TREATMENT PLANT MODEL

This chapter presents the results of a sensitivity analysis of the wastewater treatment plant model. The sensitivity equation method was used to obtain normalized sensitivity coefficients of model variables and total annual cost for all technological parameters and cost coefficients. The type B approach discussed in section 5.4.2. was used (*i.e.* the decision variables are constrained to change optimally with a variation in a parameter value). The effects of changing the primary digester rate coefficient, activated sludge thickening characteristics, aeration tank dissolved oxygen concentration, influent wastewater flowrate, and effluent standards are discussed in detail.

The presentation illustrates how the normalized sensitivity coefficients can be used to examine the complex interactions in the model and the characteristics of the optimal solution. The wastewater treatment plant considered is typical but hypothetical, so the sensitivity analysis is also hypothetical. The hypothetical quality of the problem does not prevent useful conclusions from being drawn but, in general, it does preclude any ranking of the model parameters according to how critical they are to the optimal design or cost, since the analysis is problem specific.

First, the base optimal solution for which the sensitivity analysis was conducted is presented. Restructuring of the model for the sensitivity analysis is then described. The sensitivity coefficients are then presented, and some of them are discussed in detail.

6.1. Base Optimal Solution

The values of the optimal design variables and the total cost of the base solution is given in Table 6.1. The total cost of the design is in dollar/year. This solution was obtained using the nominal parameter values listed in Table 3.2. The decision vector possesses the optimal characteristics mentioned in Chapter 4, and is the best of four solutions obtained using different feasible starting points. The decision vector and optimal cost of all four solutions are presented in Appendix B.

Table 6.1-Optimal Decision Vector and Annual Cost for Base Solution

Variable	Base Solution
Overflow Rate, Primary Clarifier (OR_p), m/hr	6.00
Sludge Age, Aeration Tank (θ_c), days	2.19
Hydraulic Retention Time (θ_{at}), days	0.16
Sludge Recycle Ratio (RR)	0.13
Solids Loading, Gravity Thickener (L_{gt}), kg/day/m^2	12.34
Digester Temperature (T_d), °C	60.00
Sludge Age, Primary Digester (θ_d), days	14.63
Solids Loading, Secondary Digester (L_d), kg/day/m^2	40.16
Vacuum Filter Yield (L_f), kg/hr/m^2	6.69
Total Cost ($/year)	500,390.00

6.2. Restructuring the Model for Sensitivity Equations

The sensitivity equations require all costraints to be equalities with zero right hand sides. Also, to obtain meaningful results, constraints must be included that reflect the bounds imposed on model variables. Upper or lower bound constraints are required, however, only for those variables that are equal to their upper or lower bounds at the optimum.

Three inequality constraints, that describe the effluent BOD_5 requirement, the effluent TSS requirement, and the minimum air flow rate for mixing in the aeration tank, were converted to equality form by the addition of slack variables. Also, constraints were added for upper bounds on the primary clarifier overflow rate (OR_p), the digestion temperature (T_d), and the solids concentration of the filter cake ($M_{t_{16}}$). Another constraint was added for a lower bound on the secondary digester supernatant flow (Q_{13}). These constraints for variables at their bound were not included in Tang's optimization model, because GRG2 handles bounds on variables implicitly in the optimization algorithm.

The values of lagrange multipliers are required by the sensitivity equations. A generalized reduced gradient algorithm such as GRG2 will yield the lagrange multipliers directly (the multipliers associated with the constraints added for variables at bound equal the reduced gradient components of those variables). If an optimization method does not yield the values of the lagrange multipliers, the multipliers can be calculated by the technique described in section 5.4.

6.3. Thickening Coefficients of Combined Sludge

Thickening of the combined waste activated and primary sludge is predicted by equation 3.2. The thickening coefficients of the combined sludge, a_c and n_c, are assumed to be functions of the mass fraction of primary sludge in the mixture, f_p. The empirical equations which Tang used to describe this relationship are:

$$a_c = a_w + a_1 f_p^{a_2} \tag{6.1}$$

$$n_c = n_w e^{n_1 f_p} \tag{6.2}$$

where n_1, a_1, and a_2 are assumed constant and a_w and n_w are thickening characteristics of the waste activated sludge.

If the above relationship is valid, the primary sludge thickening characteristics, a_p and n_p, must equal:

$$a_p = a_w + a_1 \tag{6.3}$$

$$n_p = n_w e^{n_1} \tag{6.4}$$

Tang derived these equations by setting f_p to unity in 6.1 and 6.2. Equations 6.3 and 6.4 indicate that the primary sludge thickening characteristics are a function of the thickening characteristics of the waste activated sludge. However, thickening in the primary and the final settler are observed to be independent processes. Hence, a different approach to calculating the combined sludge thickening characteristics is used in this work. If a_p and n_p are assumed to be the independent parameters and a_1 and n_1 the dependent parameters, equations 6.3 and 6.4 yield:

$$a_1 = a_p - a_w \tag{6.5}$$

$$n_1 = \ln n_p - \ln n_w \tag{6.6}$$

The thickening characteristics of the primary and waste activated sludge are parameters in the model, and a_1 and n_1, the empirical constants in equations 6.1 and 6.2, depend on the values of the waste activated and the primary sludge thickening characteristics (equations 6.5 and 6.6). It can be argued that the values of a_c and n_c should depend on the primary sludge thickening characteristics, not just the fraction of primary sludge. This change can be assumed to take place through a change in the dependent parameters, a_1 and n_1. Investigation of combined sludge thickening characteristics is needed to validate the relationship used herein.

6.4. Sensitivity Results

Normalized sensitivity coefficients for the wastewater treatment plant model are presented in Tables 6.2 and 6.3. These coefficients predict the percent change in the decision variables and total annual cost for a 1% change in the technological parameters and cost coefficients. For example, the normalized sensitivity coefficient that approximates the affect of a change in the capital recovery factor (CRF) upon the hydraulic retention time in the aeration tank (θ_{at}) has a value of -.158. This coefficient predicts that, if the CRF would *increase* by 1%, the θ_{at} would *decrease* by .158%. Similarly, if the CRF would *decrease* by 1%, the coefficient predicts that the θ_{at} would *increase* by .158%.

Table 6.2 - Normalized sensitivity coefficients of technological parameters

Parameter	Primary Clarifier Overflow Rate, OR_p	Sludge Age, θ_c	Aeration Tank Hyd. Ret. Time, θ_{at}	Recycle Ratio, RR	Solids Loading-Grav. Thickener, L_g	1 Digester Temperature, T_d	1 Digester Hyd. Ret. Time, θ_d	Solids Loading-2 Digester, L_d	Filter Yield, L_f	Annual Cost
Economic Data:										
Capital Recovery Factor	$0.0000x10^{+1}$	$-.1577x10^{-1}$	$-.1835x10^{+0}$	$0.3515x10^{+0}$	$0.2383x10^{+0}$	$0.0000x10^{+1}$	$-.4152x10^{+0}$	$0.1386x10^{+0}$	$-.4609x10^{-1}$	$0.6425x10^{+0}$
Base (1971) Cost Index	$0.0000x10^{+1}$	$0.1744x10^{-1}$	$0.1526x10^{+0}$	$-.2939x10^{+0}$	$-.2611x10^{+0}$	$0.0000x10^{+1}$	$0.3500x10^{+0}$	$-.1867x10^{+0}$	$0.6208x10^{-1}$	$-.7389x10^{+0}$
Cost Index for 1980	$0.0000x10^{+1}$	$-.1744x10^{-1}$	$-.1526x10^{+0}$	$0.2939x10^{+0}$	$0.2611x10^{+0}$	$0.0000x10^{+1}$	$-.3500x10^{+0}$	$0.1867x10^{+0}$	$-.6208x10^{-1}$	$0.7389x10^{+0}$
Operating/Maintenance Wages (dollars/hr)	$0.0000x10^{+1}$	$-.6896x10^{-3}$	$0.6323x10^{-1}$	$-.1168x10^{+0}$	$0.9739x10^{-1}$	$0.0000x10^{+1}$	$0.1276x10^{+0}$	$0.1552x10^{+0}$	$-.5160x10^{-1}$	$0.3291x10^{+0}$
Land Cost, C_L (dollars/acre)	$0.0000x10^{+1}$	$0.1225x10^{-4}$	$-.2000x10^{-4}$	$-.5768x10^{-1}$	$-.2029x10^{-1}$	$0.0000x10^{+1}$	$0.6920x10^{-1}$	$-.5827x10^{-4}$	$0.1938x10^{-4}$	$0.2760x10^{-1}$
Electricity Cost (dollars/kWhr)	$0.0000x10^{+1}$	$0.2586x10^{-4}$	$0.9386x10^{-1}$	$-.1770x10^{+0}$	$-.8894x10^{-3}$	$0.0000x10^{+1}$	$0.2875x10^{-3}$	$-.9291x10^{-3}$	$0.3090x10^{-3}$	$0.8429x10^{-1}$
Pumping Head, H (meters)	$0.0000x10^{+1}$	$0.2586x10^{-4}$	$0.9386x10^{-1}$	$-.1770x10^{+0}$	$-.8894x10^{-3}$	$0.0000x10^{+1}$	$0.2875x10^{-3}$	$-.9291x10^{-3}$	$0.3090x10^{-3}$	$0.8429x10^{-1}$
Pumping Efficiency, ε_p	$0.0000x10^{+1}$	$-.2586x10^{-4}$	$-.9386x10^{-1}$	$0.1770x10^{+0}$	$0.8894x10^{-3}$	$0.0000x10^{+1}$	$-.2875x10^{-3}$	$0.9291x10^{-3}$	$-.3090x10^{-3}$	$-.8429x10^{-1}$
Primary Sedimentation:										
Constant in Voshel-Sak Model, v_1	$0.0000x10^{+1}$	$0.8095x10^{-1}$	$-.2550x10^{+0}$	$-.1758x10^{+0}$	$0.2863x10^{+0}$	$0.0000x10^{+1}$	$0.1416x10^{+0}$	$-.6867x10^{-1}$	$0.2284x10^{-1}$	$-.5550x10^{-1}$
Constant in Voshel-Sak Model, v_2	$0.0000x10^{+1}$	$0.1163x10^{+0}$	$-.3664x10^{+0}$	$-.2526x10^{+0}$	$0.4145x10^{+0}$	$0.0000x10^{+1}$	$0.2022x10^{+0}$	$-.9539x10^{-1}$	$0.3172x10^{-1}$	$-.7974x10^{-1}$
Constant in Voshel-Sak Model, v_3	$0.0000x10^{+1}$	$-.3191x10^{-1}$	$0.1005x10^{+0}$	$0.6930x10^{-1}$	$-.1129x10^{+0}$	$0.0000x10^{+1}$	$-.5580x10^{-1}$	$0.2707x10^{-1}$	$-.9003x10^{-1}$	$0.2188x10^{-1}$
Sludge Settling Characteristics:										
Thickening Constant, a_w	$0.0000x10^{+1}$	$0.1316x10^{-1}$	$-.2769x10^{+0}$	$-.3030x10^{+0}$	$0.2185x10^{+0}$	$0.0000x10^{+1}$	$0.6535x10^{-1}$	$-.1686x10^{+0}$	$0.5606x10^{-1}$	$-.4657x10^{-1}$
Thickening Constant, a_p	$0.0000x10^{+1}$	$0.1826x10^{-1}$	$-.2116x10^{-1}$	$-.1975x10^{-1}$	$0.3458x10^{+0}$	$0.0000x10^{+1}$	$0.1073x10^{+0}$	$-.2786x10^{+0}$	$0.9264x10^{-1}$	$-.3715x10^{-1}$
Thickening Constant, a_s	$0.0000x10^{+1}$	$-.2918x10^{-1}$	$0.4013x10^{-1}$	$0.2654x10^{-1}$	$-.5487x10^{+0}$	$0.0000x10^{+1}$	$-.1702x10^{+0}$	$0.4416x10^{+0}$	$-.1469x10^{+0}$	$0.4847x10^{-1}$
Thickening Constant, n_w	$0.0000x10^{+1}$	$-.1842x10^{-1}$	$0.1081x10^{+1}$	$0.1913x10^{+1}$	$-.2127x10^{+1}$	$0.0000x10^{+1}$	$-.1017x10^{+1}$	$0.2636x10^{+1}$	$-.8766x10^{+0}$	$0.3631x10^{+0}$
Thickening Constant, n_p	$0.0000x10^{+1}$	$-.1639x10^{-1}$	$0.1876x10^{-1}$	$0.1730x10^{-1}$	$-.1991x10^{+1}$	$0.0000x10^{+1}$	$-.9644x10^{+0}$	$0.2504x10^{+1}$	$-.8327x10^{+0}$	$0.2914x10^{+0}$
Activated Sludge Kinetics:										
Growth Yield Coefficient, y (g cell/g BOD_5)	$0.0000x10^{+1}$	$-.8561x10^{+0}$	$-.1986x10^{+0}$	$-.2273x10^{+0}$	$-.2993x10^{+0}$	$0.0000x10^{+1}$	$0.1323x10^{+0}$	$0.2649x10^{+0}$	$-.8811x10^{-1}$	$-.4348x10^{-1}$
Half-Velcity Constant, K_s (g BOD_5/m³)	$0.0000x10^{+1}$	$0.8009x10^{+0}$	$0.5257x10^{+0}$	$0.4695x10^{+0}$	$0.1107x10^{-1}$	$0.0000x10^{+1}$	$-.5201x10^{-1}$	$-.1579x10^{-1}$	$0.5251x10^{-1}$	$0.5670x10^{-1}$
Maximum Specific Utilization Coeff., k_1 (day^{-1})	$0.0000x10^{+1}$	$-.1065x10^{+1}$	$-.6991x10^{+0}$	$-.6243x10^{+0}$	$-.1279x10^{-1}$	$0.0000x10^{+1}$	$0.6216x10^{-1}$	$0.2301x10^{-1}$	$-.7653x10^{-1}$	$-.7540x10^{-1}$
Endogenous Decay Coefficient, b (day^{-1})	$0.0000x10^{+1}$	$0.6516x10^{-1}$	$0.1941x10^{-1}$	$0.2044x10^{-1}$	$0.1885x10^{-1}$	$0.0000x10^{+1}$	$-.8330x10^{-1}$	$-.1667x10^{-1}$	$0.5543x10^{-1}$	$0.4614x10^{-1}$
Fraction of cells Degradable, f_d	$0.0000x10^{+1}$	$0.4413x10^{+0}$	$0.2965x10^{+0}$	$0.2648x10^{+0}$	$0.1542x10^{-1}$	$0.0000x10^{+1}$	$-.5948x10^{-1}$	$0.7602x10^{-1}$	$-.2528x10^{-1}$	$0.4324x10^{-1}$
Conversion (g BOD_L/g cell)	$0.0000x10^{+1}$	$0.5149x10^{+0}$	$0.5158x10^{+0}$	$0.4337x10^{+0}$	$-.1487x10^{+0}$	$0.0000x10^{+1}$	$0.1940x10^{+0}$	$0.1375x10^{+0}$	$-.4571x10^{-1}$	$-.4702x10^{-1}$
Conversion (g BOD_L/g BOD_5)	$0.0000x10^{+1}$	$-.5147x10^{+0}$	$-.5151x10^{+0}$	$-.4359x10^{+0}$	$0.1095x10^{+0}$	$0.0000x10^{+1}$	$-.4998x10^{-1}$	$-.1351x10^{+0}$	$0.4492x10^{-1}$	$0.6176x10^{-1}$
Secondary Sedimentation:										
Constant in Chapman Model, c_1	$0.0000x10^{+1}$	$-.4343x10^{-4}$	$0.1969x10^{-1}$	$0.1209x10^{+0}$	$-.3472x10^{-3}$	$0.0000x10^{+1}$	$0.9219x10^{-4}$	$-.3868x10^{-1}$	$0.1286x10^{-3}$	$-.6960x10^{-1}$
Constant in Chapman Model, c_2	$0.0000x10^{+1}$	$0.5932x10^{-4}$	$0.2640x10^{-1}$	$-.2200x10^{+0}$	$0.5810x10^{-3}$	$0.0000x10^{+1}$	$0.4348x10^{-4}$	$0.4051x10^{-4}$	$-.1347x10^{-4}$	$0.7484x10^{-3}$
Constant in Chapman Model, c_3	$0.0000x10^{+1}$	$0.2131x10^{-3}$	$-.1499x10^{+0}$	$-.5385x10^{+0}$	$0.2172x10^{-1}$	$0.0000x10^{+1}$	$-.6217x10^{-3}$	$0.2385x10^{-3}$	$-.7933x10^{-3}$	$0.3617x10^{-1}$
Aeration:										
Alpha Factor in Aeration	$0.0000x10^{+1}$	$-.2454x10^{-4}$	$-.4278x10^{-3}$	$0.8923x10^{-3}$	$0.3595x10^{-1}$	$0.0000x10^{+1}$	$-.1283x10^{-3}$	$0.3741x10^{-1}$	$-.1244x10^{-1}$	$-.1356x10^{+0}$
Beta Factor in Aeration	$0.0000x10^{+1}$	$-.2964x10^{-4}$	$-.5168x10^{-3}$	$0.1078x10^{-1}$	$0.4342x10^{-1}$	$0.0000x10^{+1}$	$-.1550x10^{-1}$	$0.4519x10^{-1}$	$-.1503x10^{-1}$	$-.1638x10^{+0}$
DO Concentration in Aeration Tank, DO_{at} (g/m³)	$0.0000x10^{+1}$	$0.5104x10^{-1}$	$0.8897x10^{-4}$	$-.1856x10^{-3}$	$-.7477x10^{-3}$	$0.0000x10^{+1}$	$0.2668x10^{-1}$	$-.7782x10^{-3}$	$0.2588x10^{-3}$	$0.2821x10^{-1}$
DO Saturation Concentration, C_s (g/m³)	$0.0000x10^{+1}$	$-.5104x10^{-1}$	$-.8897x10^{-4}$	$0.1856x10^{-3}$	$0.7477x10^{-3}$	$0.0000x10^{+1}$	$-.2668x10^{-1}$	$0.7782x10^{-3}$	$-.2588x10^{-3}$	$-.2821x10^{-1}$
Temperature of Mixed Liquor, T_L (°C)	$0.0000x10^{+1}$	$-.1164x10^{-4}$	$-.2029x10^{-3}$	$0.4233x10^{-3}$	$0.1705x10^{-1}$	$0.0000x10^{+1}$	$-.6084x10^{-3}$	$0.1775x10^{-1}$	$-.5902x10^{-1}$	$-.6433x10^{-1}$
Oxygen Transfer Efficiency, OTE	$0.0000x10^{+1}$	$-.2454x10^{-4}$	$-.4278x10^{-3}$	$0.8923x10^{-3}$	$0.3595x10^{-1}$	$0.0000x10^{+1}$	$-.1283x10^{-3}$	$0.3741x10^{-1}$	$-.1244x10^{-1}$	$-.1356x10^{+0}$

Table 6.2 (continued)

Parameter	Primary Clarifier Overflow Rate, OR,	Activated Sludge Sludge Age, θ_c	Aeration Tank Hyd. Ret. Time, θ_{at}	Recycle Ratio, RR	Solids Loading-Grav. Thickener, L_g	1 Digester Temperature, T_d	1 Digester Hyd. Ret. Time, θ_d	Solids Loading-2 Digester, L_d	Filter Yield, L_f	Annual Cost
Density of Air, ρ_{air} (kg/m³)	$0.0000\times10^{+1}$	$-.2454\times10^{-4}$	$-.4278\times10^{-7}$	0.8923×10^{-3}	0.3595×10^{-2}	$0.0000\times10^{+1}$	$-.1283\times10^{-1}$	0.3741×10^{-2}	$-.1244\times10^{-2}$	$-.1356\times10^{+0}$
Weight Fraction of Oxygen in Air, γ	$0.0000\times10^{+1}$	$-.2454\times10^{-4}$	$-.4278\times10^{-3}$	0.8923×10^{-3}	0.3595×10^{-2}	$0.0000\times10^{+1}$	$-.1283\times10^{-1}$	0.3741×10^{-2}	$-.1244\times10^{-2}$	$-.1356\times10^{+0}$
Mixing Requirement, η (m³air/m³/min)	$0.0000\times10^{+1}$	$0.0000\times10^{+1}$	$0.0000\times10^{+1}$	$0.0000\times10^{+1}$	$0.0000\times10^{+1}$	$0.0000\times10^{+1}$	$0.0000\times10^{+1}$	$0.0000\times10^{+1}$	$0.0000\times10^{+1}$	$0.0000\times10^{+1}$
Gravity Thickening:										
TSS of Thickener Supernatant, M_{t2e} (kg/m³)	$0.0000\times10^{+1}$	$-.1875\times10^{-1}$	0.1716×10^{-2}	$-.1995\times10^{-1}$	0.1401×10^{-2}	$0.0000\times10^{+1}$	$-.5286\times10^{-3}$	0.6940×10^{-3}	$-.2308\times10^{-4}$	0.2146×10^{-3}
Anaerobic Digestion:										
Coefficient in Reaction Rate Expression, R_1	$0.0000\times10^{+1}$	$-.2202\times10^{-1}$	0.3256×10^{-1}	0.1485×10^{-1}	$0.2894\times10^{+1}$	$0.0000\times10^{+1}$	$-.1469\times10^{+3}$	$0.6124\times10^{+1}$	$-.2037\times10^{+1}$	$-.1445\times10^{+1}$
Coefficient in Reaction Rate Expression, R_2	$0.0000\times10^{+1}$	$-.2158\times10^{-1}$	0.3191×10^{-1}	0.1455×10^{-1}	$0.2836\times10^{+1}$	$0.0000\times10^{+1}$	$-.1440\times10^{+1}$	$0.6001\times10^{+2}$	$-.1996\times10^{+1}$	$-.1416\times10^{+1}$
Temperature of Digester Influent, T_e (°C)	$0.0000\times10^{+1}$	$-.1013\times10^{-1}$	0.1441×10^{-2}	0.8336×10^{-3}	$0.1492\times10^{+0}$	$0.0000\times10^{+1}$	$-.5879\times10^{-1}$	$0.1535\times10^{+0}$	$-.5104\times10^{-1}$	$-.1814\times10^{-1}$
Methane Production (m³/kg BOD_L)	$0.0000\times10^{+1}$	0.2091×10^{-7}	0.2655×10^{-3}	$-.1245\times10^{-1}$	$-.3552\times10^{-1}$	$0.0000\times10^{+1}$	$0.1427\times10^{+0}$	0.6122×10^{-2}	$-.2036\times10^{-1}$	$-.1209\times10^{+0}$
Average Ambient Temperature, T_a (°C)	$0.0000\times10^{+1}$	$-.2769\times10^{-4}$	0.4101×10^{-4}	0.1847×10^{-4}	0.3617×10^{-1}	$0.0000\times10^{+1}$	0.1109×10^{-1}	0.7866×10^{-2}	$-.2616\times10^{-1}$	$-.1080\times10^{-1}$
Efficiency of Heat Exchanger, e	$0.0000\times10^{+1}$	$-.2165\times10^{-1}$	0.3086×10^{-2}	0.1760×10^{-2}	$0.3166\times10^{+0}$	$0.0000\times10^{+1}$	$-.6215\times10^{-1}$	$0.3463\times10^{+0}$	$-.1152\times10^{+0}$	$-.4168\times10^{-1}$
Heat Conduction Coefficient, U (W/m²-°C)	$0.0000\times10^{+1}$	0.1384×10^{-1}	$-.2051\times10^{-3}$	$-.9236\times10^{-4}$	$-.1809\times10^{-1}$	$0.0000\times10^{+1}$	$-.5543\times10^{-1}$	$-.3933\times10^{-1}$	0.1308×10^{-1}	0.5400×10^{-2}
Outside Surface Area and Volume Ratio for Digester, a	$0.0000\times10^{+1}$	0.1384×10^{-1}	$-.2051\times10^{-1}$	$-.9236\times10^{-4}$	$-.1809\times10^{-1}$	$0.0000\times10^{+1}$	$-.5543\times10^{-1}$	$-.3933\times10^{-1}$	0.1308×10^{-1}	0.5400×10^{-1}
Worth of Digester Gas (dollars/10⁶ kJ)	$0.0000\times10^{+1}$	0.2374×10^{-1}	$-.2821\times10^{-2}$	$-.3005\times10^{-2}$	$-.3521\times10^{+0}$	$0.0000\times10^{+1}$	$0.2049\times10^{+0}$	$-.3402\times10^{+0}$	$0.1131\times10^{+0}$	$-.7920\times10^{-1}$
Soluble BOD_5 in Digester Supernatant, S_{12} (g/m³)	$0.0000\times10^{+1}$	0.2094×10^{-1}	0.4475×10^{-2}	0.3595×10^{-2}	$-.2336\times10^{-1}$	$0.0000\times10^{+1}$	0.8659×10^{-2}	$-.1923\times10^{-1}$	0.6397×10^{-2}	0.2247×10^{-1}
Factor Accounting for Effect of Rising Gas on Thickening in Secondary Digester, δ	$0.0000\times10^{+1}$	0.1311×10^{-2}	$-.1822\times10^{-2}$	$-.1146\times10^{-2}$	$-.1895\times10^{+0}$	$0.0000\times10^{+1}$	$-.2233\times10^{-1}$	$0.7730\times10^{+0}$	0.7549×10^{-2}	$-.3197\times10^{-1}$
Thickening Constant for Digested Sludge, a_d	$0.0000\times10^{+1}$	0.1311×10^{-2}	$-.1822\times10^{-3}$	$-.1146\times10^{-2}$	$-.1895\times10^{+0}$	$0.0000\times10^{+1}$	$-.2233\times10^{-1}$	$0.7730\times10^{+0}$	0.7549×10^{-1}	$-.3197\times10^{-1}$
Thickening Constant for Digested Sludge, n_d	$0.0000\times10^{+1}$	$-.1397\times10^{-1}$	0.1948×10^{-1}	0.1213×10^{-1}	$0.2025\times10^{+1}$	$0.0000\times10^{+1}$	$0.1038\times10^{+0}$	$-.5010\times10^{+1}$	$-.7919\times10^{+0}$	$0.2363\times10^{+0}$
TSS of Digester Supernatant, M_{t13} (kg/m³)	$0.0000\times10^{+1}$	$-.5815\times10^{-7}$	0.4483×10^{-7}	0.2725×10^{-7}	$-.9366\times10^{-7}$	$0.0000\times10^{+1}$	$-.2223\times10^{-4}$	0.3219×10^{-6}	$-.1071\times10^{-6}$	$-.1422\times10^{-7}$
Height of Digester (m)	$0.0000\times10^{+1}$	$-.1311\times10^{-1}$	0.1822×10^{-2}	0.1146×10^{-2}	$0.1895\times10^{+0}$	$0.0000\times10^{+1}$	0.2233×10^{-1}	$0.2270\times10^{+0}$	$-.7549\times10^{-2}$	0.3197×10^{-1}
Vacuum Filtration:										
Form Time per Cycle Time, X	$0.0000\times10^{+1}$	$-.6633\times10^{-3}$	0.9510×10^{-3}	0.5323×10^{-1}	0.9838×10^{-1}	$0.0000\times10^{+1}$	$-.5770\times10^{-1}$	0.9490×10^{-1}	$0.4684\times10^{+0}$	$-.2778\times10^{-1}$
Pressure applied on Vacuum Filter, P (Nt/m²)	$0.0000\times10^{+1}$	$-.6633\times10^{-3}$	0.9510×10^{-1}	0.5323×10^{-2}	0.9838×10^{-1}	$0.0000\times10^{+1}$	$-.5770\times10^{-1}$	0.9490×10^{-1}	$0.4684\times10^{+0}$	$-.2778\times10^{-1}$
Viscosity of Filtrate, μ (Nt-sec/m²)	$0.0000\times10^{+1}$	0.6633×10^{-3}	$-.9511\times10^{-3}$	$-.5323\times10^{-3}$	$-.9839\times10^{-1}$	$0.0000\times10^{+1}$	0.5770×10^{-1}	$-.9491\times10^{-1}$	$-.4685\times10^{+0}$	0.2779×10^{-1}
Cycle Time, t_c (min)	$0.0000\times10^{+1}$	0.6633×10^{-3}	$-.9510\times10^{-2}$	$-.5323\times10^{-3}$	$-.9838\times10^{-1}$	$0.0000\times10^{+1}$	0.5770×10^{-1}	$-.9490\times10^{-2}$	$-.4684\times10^{+0}$	0.2778×10^{-1}
Specific Resistance of Sludge, r_s (m/kg)	$0.0000\times10^{+1}$	0.6633×10^{-3}	$-.9510\times10^{-3}$	$-.5323\times10^{-3}$	$-.9838\times10^{-1}$	$0.0000\times10^{+1}$	0.5770×10^{-1}	$-.9490\times10^{-1}$	$-.4684\times10^{+0}$	0.2778×10^{-1}
TSS of Filtrate, M_{t15} (kg/m³)	$0.0000\times10^{+1}$	$-.1141\times10^{-1}$	0.8851×10^{-2}	0.5383×10^{-2}	$-.6461\times10^{-2}$	$0.0000\times10^{+1}$	$-.4077\times10^{-1}$	$-.9284\times10^{-1}$	$-.2621\times10^{-1}$	0.5362×10^{-1}
Effluent Standards:										
BOD_5 Concenration (mg/l)	$0.0000\times10^{+1}$	$-.1259\times10^{+1}$	$-.9051\times10^{+0}$	$-.7980\times10^{+0}$	0.3712×10^{-1}	$0.0000\times10^{+1}$	$-.1650\times10^{-1}$	$-.3694\times10^{-1}$	0.1229×10^{-1}	$-.1380\times10^{+0}$
TSS Concentration (mg/l)	$0.0000\times10^{+1}$	$0.4299\times10^{+0}$	$0.4056\times10^{+0}$	$0.9039\times10^{+0}$	$0.1041\times10^{+0}$	$0.0000\times10^{+1}$	0.6072×10^{-3}	0.3787×10^{-1}	$-.1259\times10^{-1}$	$-.5277\times10^{-1}$
Plant Influent Conditions:										
Wastewater Flow, Q_o (m³/hr)	$0.0000\times10^{+1}$	0.7977×10^{-1}	0.9028×10^{-2}	$-.1982\times10^{-1}$	$-.1204\times10^{+0}$	$0.0000\times10^{+1}$	$0.1358\times10^{+0}$	$-.9458\times10^{-1}$	0.3145×10^{-1}	$0.6069\times10^{+0}$
Soluble BOD_5, S_o (mg/l)	$0.0000\times10^{+1}$	$0.1505\times10^{+0}$	$0.3603\times10^{+0}$	$0.2860\times10^{+0}$	$-.2019\times10^{+0}$	$0.0000\times10^{+1}$	0.8915×10^{-1}	$0.1788\times10^{+0}$	$-.5947\times10^{-1}$	$0.1727\times10^{+0}$
Active Biomass, M_{ao} (mg/l)	$0.0000\times10^{+1}$	0.1312×10^{-1}	$-.1245\times10^{-1}$	$-.1354\times10^{-2}$	$-.1656\times10^{-1}$	$0.0000\times10^{+1}$	0.1650×10^{-1}	0.2092×10^{-1}	$-.6958\times10^{-2}$	0.2083×10^{-1}
Degradable Suspended Solids, M_{do} (mg/l)	$0.0000\times10^{+1}$	0.9595×10^{-1}	$0.1753\times10^{+0}$	$0.1409\times10^{+0}$	$-.1170\times10^{+0}$	$0.0000\times10^{+1}$	$0.2355\times10^{+0}$	$0.2351\times10^{+0}$	$-.7820\times10^{-1}$	$0.1023\times10^{+0}$
Inert Suspended Solids, M_{io} (mg/l)	$0.0000\times10^{+1}$	$-.9803\times10^{-1}$	0.7564×10^{-1}	0.4557×10^{-1}	$-.1493\times10^{+0}$	$0.0000\times10^{+1}$	$0.1480\times10^{+0}$	$0.1750\times10^{+0}$	$-.5820\times10^{-1}$	0.3078×10^{-1}
Fixed Suspended Solids, M_{fo} (mg/l)	$0.0000\times10^{+1}$	$-.1110\times10^{+0}$	0.8716×10^{-1}	0.5239×10^{-1}	$0.2914\times10^{+0}$	$0.0000\times10^{+1}$	$-.2950\times10^{+0}$	$-.7067\times10^{+0}$	$0.2350\times10^{+0}$	$0.2739\times10^{+0}$

Table 6.3 - Normalized sensitivity coefficients of cost coefficients

Cost Coefficient	Primary Clarifier Overflow Rate, OR_p	Sludge Age, θ_c	Aeration Tank Hyd. Ret. Time, θ_{at}	Recycle Ratio, RR	Solids Loading-Grav. Thickener, L_g	1 Digester Temperature, T_d	1 Digester Hyd. Ret. Time, θ_d	Solids Loading-2 Digester, L_d	Filter Yield, L_f	Annual Cost
Cost Coefficient-Operation	0.0000x10^{+1}	0.2035x10^{-6}	0.2289x10^{-3}	-.4322x10^{-3}	-.2256x10^{-4}	0.0000x10^{+1}	0.1207x10^{-5}	-.2573x10^{-4}	0.8559x10^{-5}	0.6492x10^{-4}
Cost Coefficient-Maintenance	0.0000x10^{+1}	0.1451x10^{-6}	0.1633x10^{-3}	-.3083x10^{-3}	-.1609x10^{-4}	0.0000x10^{+1}	0.8610x10^{-6}	-.1835x10^{-4}	0.6104x10^{-6}	0.4744x10^{-4}
Cost of Materials (constant)	0.0000x10^{+1}	0.0000x10^{+1}	0.0000x10^{+1}	0.0000x10^{+1}	0.0000x10^{+1}	0.0000x10^{+1}	0.0000x10^{+1}	0.0000x10^{+1}	0.0000x10^{+1}	0.1441x10^{-3}
Gravity Thickener:										
Cost Coefficient-Capital Cost	0.0000x10^{+1}	-.2857x10^{-2}	0.3893x10^{-2}	0.2669x10^{-3}	0.4208x10^{+0}	0.0000x10^{+1}	-.1667x10^{+0}	0.4325x10^{+0}	-.1438x10^{+0}	0.4492x10^{-1}
Cost Exponent-Capital Cost	0.0000x10^{+1}	-.1646x10^{-1}	0.2243x10^{-1}	0.1538x10^{-1}	0.2425x10^{+1}	0.0000x10^{+1}	-.9606x10^{+0}	0.2492x10^{+1}	-.8289x10^{+0}	0.2138x10^{+0}
Cost Coefficient-Operation	0.0000x10^{+1}	-.6169x10^{-3}	0.8406x10^{-3}	0.5763x10^{-3}	0.9086x10^{-1}	0.0000x10^{+1}	-.3599x10^{-1}	0.9338x10^{-1}	-.3106x10^{-1}	0.1245x10^{-1}
Cost Exponent-Operation	0.0000x10^{+1}	-.2907x10^{-2}	0.3960x10^{-2}	0.2715x10^{-1}	0.4281x10^{+0}	0.0000x10^{+1}	-.1696x10^{+0}	0.4400x10^{+0}	-.1463x10^{+0}	0.4620x10^{-1}
Cost Coefficient-Maintenance	0.0000x10^{+1}	-.3320x10^{-3}	0.4524x10^{-3}	0.3102x10^{-3}	0.4890x10^{-1}	0.0000x10^{+1}	-.1937x10^{-1}	0.5026x10^{-1}	-.1671x10^{-1}	0.6703x10^{-2}
Cost Exponent-Maintenance	0.0000x10^{+1}	-.1564x10^{-1}	0.2131x10^{-2}	0.1461x10^{-2}	0.2304x10^{+0}	0.0000x10^{+1}	-.9126x10^{-1}	0.2368x10^{+0}	-.7875x10^{-1}	0.2486x10^{-1}
Cost Coefficient-Materials	0.0000x10^{+1}	-.2854x10^{-3}	0.3889x10^{-3}	0.2666x10^{-3}	0.4204x10^{-1}	0.0000x10^{+1}	-.1665x10^{-1}	0.4320x10^{-1}	-.1437x10^{-1}	0.4546x10^{-2}
Cost Exponent-Materials	0.0000x10^{+1}	-.1627x10^{-2}	0.2217x10^{-1}	0.1520x10^{-2}	0.2397x10^{+0}	0.0000x10^{+1}	-.9493x10^{-1}	0.2463x10^{+0}	-.8191x10^{-1}	0.2136x10^{-1}
Primary Anaerobic Digester:										
Cost Coefficient-Capital Cost	0.0000x10^{+1}	0.1240x10^{-2}	-.1837x10^{-1}	-.8274x10^{-3}	-.1620x10^{+0}	0.0000x10^{+1}	-.4965x10^{+0}	-.3523x10^{+0}	0.1172x10^{+0}	0.8226x10^{-1}
Cost Exponent-Capital Cost	0.0000x10^{+1}	0.6615x10^{-2}	-.9798x10^{-1}	-.4413x10^{-2}	-.8642x10^{+0}	0.0000x10^{+1}	-.2648x10^{+1}	-.1879x10^{+1}	0.6250x10^{+0}	0.3561x10^{+0}
Cost Coefficient-Operation	0.0000x10^{+1}	0.7547x10^{-4}	-.1118x10^{-3}	-.5035x10^{-4}	-.9859x10^{-1}	0.0000x10^{+1}	-.3022x10^{-1}	-.2144x10^{-1}	0.7131x10^{-2}	0.1481x10^{-1}
Cost Exponent-Operation	0.0000x10^{+1}	0.1863x10^{-3}	-.2760x10^{-3}	-.1243x10^{-3}	-.2434x10^{-1}	0.0000x10^{+1}	-.7461x10^{-1}	-.5294x10^{-1}	0.1761x10^{-1}	0.2174x10^{-1}
Cost Coefficient-Maintenance	0.0000x10^{+1}	0.5019x10^{-4}	-.7435x10^{-4}	-.3349x10^{-4}	-.6557x10^{-2}	0.0000x10^{+1}	-.2010x10^{-1}	-.1426x10^{-1}	0.4742x10^{-2}	0.9383x10^{-1}
Cost Exponent-Maintenance	0.0000x10^{+1}	0.1276x10^{-3}	-.1890x10^{-3}	-.8515x10^{-4}	-.1667x10^{-1}	0.0000x10^{+1}	-.5109x10^{-1}	-.3626x10^{-1}	0.1206x10^{-1}	0.1446x10^{-1}
Cost Coefficient-Materials	0.0000x10^{+1}	0.9722x10^{-4}	-.1440x10^{-3}	-.6486x10^{-4}	-.1270x10^{-1}	0.0000x10^{+1}	-.3892x10^{-1}	-.2762x10^{-1}	0.9185x10^{-2}	0.1030x10^{-1}
Cost Exponent-Materials	0.0000x10^{+1}	0.3614x10^{-3}	-.5354x10^{-3}	-.2411x10^{-3}	-.4722x10^{-1}	0.0000x10^{+1}	-.1447x10^{+0}	-.1027x10^{+0}	0.3415x10^{-1}	0.2797x10^{-1}
Secondary Anaerobic Digester:										
Cost Coefficient-Capital Cost	0.0000x10^{+1}	-.1990x10^{-1}	0.2767x10^{-1}	0.1741x10^{-2}	0.2877x10^{+0}	0.0000x10^{+1}	0.3391x10^{-1}	0.3447x10^{+0}	-.1146x10^{+0}	0.4319x10^{-1}
Cost Exponent-Capital Cost	0.0000x10^{+1}	-.9322x10^{-2}	0.1296x10^{-1}	0.8154x10^{-2}	0.1347x10^{+1}	0.0000x10^{+1}	0.1588x10^{+0}	0.1614x10^{+1}	-.5369x10^{+0}	0.1591x10^{+0}
Cost Coefficient-Operation	0.0000x10^{+1}	-.1861x10^{-3}	0.2588x10^{-3}	0.1628x10^{-3}	0.2690x10^{-1}	0.0000x10^{+1}	0.3171x10^{-2}	0.3223x10^{-1}	-.1072x10^{-1}	0.1191x10^{-1}
Cost Exponent-Operation	0.0000x10^{+1}	-.4185x10^{-3}	0.5819x10^{-3}	0.3661x10^{-3}	0.6050x10^{-1}	0.0000x10^{+1}	0.7131x10^{-2}	0.7248x10^{-1}	-.2410x10^{-2}	0.1487x10^{-1}
Cost Coefficient-Maintenance	0.0000x10^{+1}	-.1224x10^{-3}	0.1702x10^{-3}	0.1071x10^{-3}	0.1770x10^{-1}	0.0000x10^{+1}	0.2086x10^{-2}	0.2120x10^{-2}	-.7051x10^{-1}	0.7460x10^{-2}
Cost Exponent-Maintenance	0.0000x10^{+1}	-.2829x10^{-3}	0.3934x10^{-3}	0.2475x10^{-3}	0.4090x10^{-1}	0.0000x10^{+1}	0.4821x10^{-2}	0.4900x10^{-1}	-.1630x10^{-1}	0.9783x10^{-1}
Cost Coefficient-Materials	0.0000x10^{+1}	-.1988x10^{-3}	0.2764x10^{-3}	0.1739x10^{-3}	0.2874x10^{-1}	0.0000x10^{+1}	0.3387x10^{-2}	0.3443x10^{-1}	-.1145x10^{-1}	0.6877x10^{-2}
Cost Exponent-Materials	0.0000x10^{+1}	-.6581x10^{-3}	0.9150x10^{-3}	0.5756x10^{-3}	0.9512x10^{-2}	0.0000x10^{+1}	0.1121x10^{-2}	0.1140x10^{+0}	-.3790x10^{-1}	0.1589x10^{-1}
Vacuum Filter:										
Cost Coefficient-Capital Cost	0.0000x10^{+1}	0.1868x10^{-2}	-.2679x10^{-1}	-.1499x10^{-2}	-.2771x10^{+0}	0.0000x10^{+1}	0.1625x10^{+0}	-.2673x10^{+0}	0.8891x10^{-1}	0.7826x10^{-1}
Cost Exponent-Capital Cost	0.0000x10^{+1}	0.5133x10^{-1}	-.7359x10^{-1}	-.4119x10^{-2}	-.7613x10^{+0}	0.0000x10^{+1}	0.4465x10^{+0}	-.7344x10^{+0}	0.2442x10^{+0}	0.1369x10^{+0}
Cost Coefficient-Materials (1st)	0.0000x10^{+1}	0.7715x10^{-4}	-.1260x10^{-1}	-.3634x10^{-4}	-.1278x10^{-1}	0.0000x10^{+1}	0.4360x10^{-1}	-.3671x10^{-3}	0.1221x10^{-3}	0.2448x10^{-1}
Cost Exponent-Materials (1st)	0.0000x10^{+1}	0.3161x10^{-3}	-.5163x10^{-3}	-.1489x10^{-3}	-.5237x10^{-1}	0.0000x10^{+1}	0.1786x10^{+0}	-.1504x10^{-1}	0.5002x10^{-3}	0.7584x10^{-1}
Cost Coefficient-Materials (2nd)	0.0000x10^{+1}	0.1423x10^{-3}	-.2324x10^{-3}	-.6701x10^{-4}	-.2357x10^{-1}	0.0000x10^{+1}	0.8039x10^{-1}	-.6769x10^{-3}	0.2251x10^{-3}	0.3727x10^{-1}
Cost Exponent-Materials (2nd)	0.0000x10^{+1}	0.6759x10^{-3}	-.1104x10^{-2}	-.3183x10^{-3}	-.1120x10^{+0}	0.0000x10^{+1}	0.3819x10^{+0}	-.3216x10^{-1}	0.1070x10^{-2}	0.1399x10^{+0}
Cost Coefficient-Operation	0.0000x10^{+1}	0.1137x10^{-3}	-.1857x10^{-2}	-.5355x10^{-4}	-.1884x10^{-1}	0.0000x10^{+1}	0.6425x10^{-1}	-.5410x10^{-1}	0.1799x10^{-3}	0.4415x10^{-1}
Cost Exponent-Operation	0.0000x10^{+1}	0.4013x10^{-3}	-.6556x10^{-3}	-.1890x10^{-3}	-.6649x10^{-1}	0.0000x10^{+1}	0.2268x10^{+0}	-.1909x10^{-2}	0.6350x10^{-2}	0.1117x10^{+0}
Cost Coefficient-Maintenance	0.0000x10^{+1}	0.1278x10^{-4}	-.2087x10^{-4}	-.6019x10^{-5}	-.2117x10^{-1}	0.0000x10^{+1}	0.7221x10^{-1}	-.6080x10^{-4}	0.2022x10^{-4}	0.5995x10^{-1}
Cost Exponent-Maintenance	0.0000x10^{+1}	0.3953x10^{-4}	-.6458x10^{-4}	-.1862x10^{-4}	-.6550x10^{-2}	0.0000x10^{+1}	0.2234x10^{-1}	-.1881x10^{-3}	0.6256x10^{-4}	0.1256x10^{-1}
Sludge Disposal:										
Cost Coefficient-Capital Cost	0.0000x10^{+1}	0.6843x10^{-4}	-.1118x10^{-5}	-.3223x10^{-4}	-.1134x10^{-1}	0.0000x10^{+1}	0.3867x10^{-2}	-.3256x10^{-3}	0.1083x10^{-5}	0.2083x10^{-1}
Cost Exponent-Capital Cost	0.0000x10^{+1}	0.3557x10^{-4}	-.5811x10^{-4}	-.1675x10^{-4}	-.5894x10^{-1}	0.0000x10^{+1}	0.2010x10^{-1}	-.1693x10^{-3}	0.5629x10^{-4}	-.9975x10^{-2}
Cost Coefficient-Operation	0.0000x10^{+1}	0.2745x10^{-3}	-.4484x10^{-3}	-.1293x10^{-3}	-.4547x10^{-1}	0.0000x10^{+1}	0.1551x10^{+0}	-.1306x10^{-1}	0.4343x10^{-3}	0.9269x10^{-1}
Cost Exponent-Operation	0.0000x10^{+1}	0.1557x10^{-3}	-.2543x10^{-3}	-.7332x10^{-4}	-.2579x10^{-1}	0.0000x10^{+1}	0.8797x10^{-1}	-.7407x10^{-3}	0.2463x10^{-3}	-.4000x10^{-1}

Table 6.3 (continued)

Cost Coefficient	Primary Clarifier Overflow Rate, OR_p	Sludge Age, θ_c	Aeration Tank Hyd. Ret. Time, θ_{at}	Recycle Ratio, RR	Solids Loading-Grav. Thickener, L_g	1 Digester Temperature, T_d	1 Digester Hyd. Ret. Time, θ_d	Solids Loading-2 Digester, L_d	Filter Yield, L_f	Annual Cost
Primary Sedimentation:										
Cost Coefficient-Capital Cost	$0.0000x10^{+1}$	$0.4133x10^{-6}$	$0.5345x10^{-3}$	$-.1009x10^{-3}$	$-.4361x10^{-4}$	$0.0000x10^{+1}$	$-.4613x10^{-6}$	$-.5066x10^{-4}$	$0.1685x10^{-4}$	$0.2714x10^{-1}$
Cost Exponent-Capital Cost	$0.0000x10^{+1}$	$0.2173x10^{-5}$	$0.2810x10^{-1}$	$-.5304x10^{-2}$	$-.2292x10^{-3}$	$0.0000x10^{+1}$	$-.2425x10^{-5}$	$-.2663x10^{-3}$	$0.8855x10^{-4}$	$0.1155x10^{+0}$
Cost Coefficient-Operation	$0.0000x10^{+1}$	$0.5124x10^{-7}$	$0.6626x10^{-4}$	$-.1251x10^{-3}$	$-.5405x10^{-5}$	$0.0000x10^{+1}$	$-.5718x10^{-7}$	$-.6279x10^{-5}$	$0.2088x10^{-5}$	$0.8634x10^{-1}$
Cost Exponent-Operation	$0.0000x10^{+1}$	$0.1362x10^{-6}$	$0.1761x10^{-3}$	$-.3325x10^{-3}$	$-.1437x10^{-4}$	$0.0000x10^{+1}$	$-.1520x10^{-6}$	$-.1669x10^{-4}$	$0.5552x10^{-5}$	$0.1432x10^{-1}$
Cost Coefficient-Maintenance	$0.0000x10^{+1}$	$0.1132x10^{-7}$	$0.1464x10^{-4}$	$-.2764x10^{-4}$	$-.1194x10^{-5}$	$0.0000x10^{+2}$	$-.1263x10^{-7}$	$-.1387x10^{-5}$	$0.4614x10^{-6}$	$0.4088x10^{-3}$
Cost Exponent-Maintenance	$0.0000x10^{+1}$	$0.2008x10^{-7}$	$0.2597x10^{-4}$	$-.4902x10^{-4}$	$-.2119x10^{-5}$	$0.0000x10^{+1}$	$-.2241x10^{-7}$	$-.2461x10^{-6}$	$0.8185x10^{-6}$	$0.3163x10^{-2}$
Cost Coefficient-Materials	$0.0000x10^{+1}$	$0.4156x10^{-7}$	$0.5375x10^{-4}$	$-.1015x10^{-3}$	$-.4385x10^{-5}$	$0.0000x10^{+2}$	$-.4638x10^{-7}$	$-.5094x10^{-5}$	$0.1694x10^{-5}$	$0.2764x10^{-2}$
Cost Exponent-Materials	$0.0000x10^{+1}$	$0.2162x10^{-6}$	$0.2796x10^{-3}$	$-.5277x10^{-3}$	$-.2281x10^{-4}$	$0.0000x10^{+2}$	$-.2412x10^{-6}$	$-.2649x10^{-4}$	$0.8811x10^{-5}$	$0.1161x10^{-1}$
Primary Sludge Pumping:										
Cost Coefficient-Capital Cost	$0.0000x10^{+1}$	$0.1437x10^{-4}$	$0.1032x10^{-1}$	$-.2449x10^{-3}$	$-.2110x10^{-2}$	$0.0000x10^{+1}$	$0.7593x10^{-3}$	$-.2194x10^{-2}$	$0.7295x10^{-3}$	$0.9538x10^{-2}$
Cost Exponent-Capital Cost	$0.0000x10^{+1}$	$0.1785x10^{-4}$	$0.1282x10^{-3}$	$-.3042x10^{-3}$	$-.2620x10^{-2}$	$0.0000x10^{+2}$	$0.9431x10^{-3}$	$-.2725x10^{-2}$	$0.9061x10^{-3}$	$0.2310x10^{-3}$
Cost Coefficient-Operation	$0.0000x10^{+1}$	$0.9349x10^{-5}$	$0.6717x10^{-4}$	$-.1594x10^{-3}$	$-.1373x10^{-1}$	$0.0000x10^{+1}$	$0.4940x10^{-3}$	$-.1427x10^{-2}$	$0.4746x10^{-3}$	$0.8022x10^{-3}$
Cost Exponent-Operation	$0.0000x10^{+2}$	$0.1110x10^{-4}$	$0.7976x10^{-4}$	$-.1892x10^{-3}$	$-.1630x10^{-2}$	$0.0000x10^{+1}$	$0.5865x10^{-3}$	$-.1694x10^{-1}$	$0.5635x10^{-3}$	$0.1503x10^{-2}$
Cost Coefficient-Maintenance	$0.0000x10^{+1}$	$0.4392x10^{-5}$	$0.3156x10^{-4}$	$-.7487x10^{-4}$	$-.6448x10^{-3}$	$0.0000x10^{+2}$	$0.2321x10^{-3}$	$-.6705x10^{-3}$	$0.2230x10^{-3}$	$0.3593x10^{-2}$
Cost Exponent-Maintenance	$0.0000x10^{+2}$	$0.5254x10^{-5}$	$0.3776x10^{-4}$	$-.8957x10^{-4}$	$-.7715x10^{-3}$	$0.0000x10^{+1}$	$0.2777x10^{-3}$	$-.8021x10^{-3}$	$0.2668x10^{-3}$	$0.7059x10^{-3}$
Cost Coefficient-Materials	$0.0000x10^{+1}$	$0.4507x10^{-5}$	$0.3238x10^{-4}$	$-.7682x10^{-4}$	$-.6617x10^{-3}$	$0.0000x10^{+2}$	$0.2381x10^{-3}$	$-.6880x10^{-3}$	$0.2288x10^{-3}$	$0.2477x10^{-2}$
Cost Exponent-Materials	$0.0000x10^{+1}$	$0.5824x10^{-5}$	$0.4185x10^{-4}$	$-.9928x10^{-4}$	$-.8551x10^{-3}$	$0.0000x10^{+2}$	$0.3078x10^{-3}$	$-.8891x10^{-3}$	$0.2957x10^{-3}$	$0.7244x10^{-3}$
Aeration Tank:										
Cost Coefficient-Capital Cost	$0.0000x10^{+1}$	$-.3691x10^{-4}$	$-.4489x10^{+0}$	$0.8463x10^{+0}$	$-.8494x10^{-2}$	$0.0000x10^{+2}$	$0.2963x10^{-2}$	$-.8893x10^{-2}$	$0.2958x10^{-2}$	$0.9970x10^{-1}$
Cost Exponent-Capital Cost	$0.0000x10^{+2}$	$-.2635x10^{-3}$	$-.3204x10^{+1}$	$0.6041x10^{+1}$	$-.6062x10^{-2}$	$0.0000x10^{+2}$	$0.2115x10^{-1}$	$-.6348x10^{-1}$	$0.2111x10^{-1}$	$0.6120x10^{+0}$
Diffused Air Aeration:										
Cost Coefficient-Capital Cost	$0.0000x10^{+1}$	$0.2881x10^{-4}$	$0.5022x10^{-1}$	$-.1048x10^{-2}$	$-.4220x10^{-2}$	$0.0000x10^{+2}$	$0.1506x10^{-2}$	$-.4392x10^{-2}$	$0.1461x10^{-2}$	$0.1492x10^{+0}$
Cost Exponent-Capital Cost	$0.0000x10^{+1}$	$0.1332x10^{-3}$	$0.2322x10^{-2}$	$-.4843x10^{-2}$	$-.1951x10^{-1}$	$0.0000x10^{+1}$	$0.6962x10^{-2}$	$-.2031x10^{-1}$	$0.6753x10^{-1}$	$0.5405x10^{+0}$
Cost Coefficient-Operation	$0.0000x10^{+1}$	$0.6514x10^{-5}$	$0.1136x10^{-3}$	$-.2369x10^{-3}$	$-.9542x10^{-3}$	$0.0000x10^{+1}$	$0.3405x10^{-3}$	$-.9931x10^{-3}$	$0.3303x10^{-3}$	$0.4638x10^{-1}$
Cost Exponent-Operation	$0.0000x10^{+1}$	$0.2368x10^{-4}$	$0.4128x10^{-3}$	$-.8610x10^{-1}$	$-.3469x10^{-2}$	$0.0000x10^{+1}$	$0.1238x10^{-2}$	$-.3610x10^{-2}$	$0.1201x10^{-2}$	$0.1222x10^{+0}$
Cost Coefficient-Maintenance	$0.0000x10^{+1}$	$0.4361x10^{-5}$	$0.7602x10^{-4}$	$-.1586x10^{-3}$	$-.6388x10^{-3}$	$0.0000x10^{+1}$	$0.2280x10^{-3}$	$-.6649x10^{-3}$	$0.2211x10^{-3}$	$0.2710x10^{-1}$
Cost Exponent-Maintenance	$0.0000x10^{+1}$	$0.1753x10^{-4}$	$0.3056x10^{-3}$	$-.6374x10^{-2}$	$-.2568x10^{-2}$	$0.0000x10^{+2}$	$0.9163x10^{-3}$	$-.2672x10^{-2}$	$0.8887x10^{-3}$	$0.8182x10^{-2}$
Final Settling Tank:										
Cost Coefficient-Capital Cost	$0.0000x10^{+1}$	$0.3188x10^{-4}$	$0.1213x10^{+0}$	$-.2287x10^{+0}$	$-.9262x10^{-3}$	$0.0000x10^{+2}$	$0.3242x10^{-2}$	$-.9577x10^{-3}$	$0.3185x10^{-3}$	$0.5862x10^{-1}$
Cost Exponent-Capital Cost	$0.0000x10^{+1}$	$0.1922x10^{-2}$	$0.7310x10^{+0}$	$-.1379x10^{+1}$	$-.5582x10^{-2}$	$0.0000x10^{+1}$	$0.1954x10^{-2}$	$-.5772x10^{-2}$	$0.1920x10^{-2}$	$0.2947x10^{+0}$
Cost Coefficient-Operation	$0.0000x10^{+1}$	$0.6495x10^{-5}$	$0.2470x10^{-1}$	$-.4659x10^{-1}$	$-.1887x10^{-3}$	$0.0000x10^{+1}$	$0.6604x10^{-4}$	$-.1951x10^{-3}$	$0.6488x10^{-4}$	$0.1533x10^{-1}$
Cost Exponent-Operation	$0.0000x10^{+2}$	$0.3194x10^{-4}$	$0.1215x10^{+0}$	$-.2291x10^{+0}$	$-.9277x10^{-3}$	$0.0000x10^{+1}$	$0.3247x10^{-3}$	$-.9593x10^{-3}$	$0.3190x10^{-3}$	$0.6004x10^{-2}$
Cost Coefficient-Maintenance	$0.0000x10^{+1}$	$0.3495x10^{-5}$	$0.1330x10^{-1}$	$-.2507x10^{-1}$	$-.1015x10^{-3}$	$0.0000x10^{+1}$	$0.3554x10^{-4}$	$-.1050x10^{-3}$	$0.3492x10^{-4}$	$0.8249x10^{-1}$
Cost Exponent-Maintenance	$0.0000x10^{+1}$	$0.1719x10^{-4}$	$0.6538x10^{-1}$	$-.1233x10^{+0}$	$-.4993x10^{-3}$	$0.0000x10^{+1}$	$0.1748x10^{-3}$	$-.5163x10^{-3}$	$0.1717x10^{-3}$	$0.3231x10^{-1}$
Cost Coefficient-Materials	$0.0000x10^{+1}$	$0.3174x10^{-5}$	$0.1207x10^{-1}$	$-.2277x10^{-1}$	$-.9221x10^{-4}$	$0.0000x10^{+1}$	$0.3227x10^{-4}$	$-.9534x10^{-4}$	$0.3171x10^{-4}$	$0.5913x10^{-2}$
Cost Exponent-Materials	$0.0000x10^{+1}$	$0.1892x10^{-4}$	$0.7198x10^{-2}$	$-.1358x10^{+0}$	$-.5497x10^{-3}$	$0.0000x10^{+1}$	$0.1924x10^{-3}$	$-.5684x10^{-3}$	$0.1890x10^{-3}$	$0.2934x10^{-1}$
Return Sludge Pumping:										
Cost Coefficient-Capital Cost	$0.0000x10^{+2}$	$0.3488x10^{-4}$	$0.1325x10^{+0}$	$-.2498x10^{+0}$	$-.1021x10^{-2}$	$0.0000x10^{+2}$	$0.3873x10^{-3}$	$-.1046x10^{-2}$	$0.3480x10^{-1}$	$0.2147x10^{-1}$
Cost Exponent-Capital Cost	$0.0000x10^{+1}$	$0.1328x10^{-3}$	$0.5044x10^{+0}$	$-.9513x10^{+0}$	$-.3889x10^{-2}$	$0.0000x10^{+1}$	$0.1475x10^{-2}$	$-.3984x10^{-2}$	$0.1325x10^{-2}$	$0.6025x10^{-2}$
Cost Coefficient-Operation	$0.0000x10^{+1}$	$0.3626x10^{-5}$	$0.1377x10^{-1}$	$-.2596x10^{-1}$	$-.1062x10^{-3}$	$0.0000x10^{+1}$	$0.4025x10^{-4}$	$-.1087x10^{-3}$	$0.3617x10^{-4}$	$0.1181x10^{-1}$
Cost Coefficient-Maintenance	$0.0000x10^{+1}$	$0.2586x10^{-5}$	$0.9819x10^{-2}$	$-.1852x10^{-1}$	$-.7571x10^{-4}$	$0.0000x10^{+1}$	$0.2871x10^{-4}$	$-.7756x10^{-4}$	$0.2579x10^{-4}$	$0.8424x10^{-3}$
Cost Coefficient-Materials	$0.0000x10^{+1}$	$0.4877x10^{-5}$	$0.1852x10^{-2}$	$-.3493x10^{-1}$	$-.1428x10^{-3}$	$0.0000x10^{+2}$	$0.5415x10^{-4}$	$-.1463x10^{-3}$	$0.4865x10^{-4}$	$0.3059x10^{-2}$
Cost Exponent-Materials	$0.0000x10^{+2}$	$0.1831x10^{-4}$	$0.6954x10^{-1}$	$-.1312x10^{+0}$	$-.5362x10^{-3}$	$0.0000x10^{+2}$	$0.2033x10^{-3}$	$-.5493x10^{-3}$	$0.1827x10^{-3}$	$0.8425x10^{-2}$
Recycle Pumping:										
Cost Coefficient-Capital Cost	$0.0000x10^{+1}$	$0.7574x10^{-5}$	$0.8522x10^{-2}$	$-.1609x10^{-1}$	$-.8399x10^{-3}$	$0.0000x10^{+1}$	$0.4494x10^{-4}$	$-.9580x10^{-3}$	$0.3186x10^{-3}$	$0.4673x10^{-2}$
Cost Exponent-Capital Cost	$0.0000x10^{+2}$	$0.1729x10^{-4}$	$0.1945x10^{-1}$	$-.3672x10^{-1}$	$-.1917x10^{-3}$	$0.0000x10^{+2}$	$0.1026x10^{-3}$	$-.2186x10^{-2}$	$0.7271x10^{-3}$	$0.5991x10^{-3}$

Examination of the sensitivity results shows that the overflow rate in the primary settling tank (OR_p) and the primary digester temperature (T_d) are completely insensitive to parameter variations. These decision variables are at their bounds at the base optimum, and the sensitivity equation approach will always yield this result for a variable at a bound. A nonzero sensitivity coefficient would indicate that the variable would violate its bound given an increase or decrease in the associated parameter. However, an example problem can be easily constructed to refute this result; if a parameter is perturbed, a variable previously at bound may be released from its bound at the new optimum. Unfortunately, sensitivity coefficients cannot supply this kind of information. It suffices to recognize this drawback to the sensitivity equation approach, and to investigate thoroughly any uncertain parameters thought to interact with a variable at bound.

6.4.1. Variation in Primary Digester Reaction Rate

The primary digester is modeled as a completely-mixed stirred tank reactor and stabilization of the sewage sludge is assumed to follow first order kinetics. The digester reaction rate expression, from Figure 3.2, is:

$$\log_{10} K_1 = \frac{1000(R_1)}{T_d + 273.} + R_2 \tag{6.7}$$

K_1 is the first order digester reaction rate and R_1 and R_2 are technological parameters. R_1 and R_2 have nominal values of -3.3333 and 9.81046, respectively.

Wise [1980] compiled the data in Figure 3.2 from several different sources; most reaction rates are for substrates other than sewage sludge. Thus, there is considerable uncertainty associated with the parameters, R_1 and R_2, in the rate expression. Because of this uncertainty, the sensitivity of the optimal solution to a change in R_1 and R_2 should be examined.

The sensitivity analysis presented below only considers a change in R_1. However, the sensitivity coefficients for R_1 and R_2 are nearly identical; the discussion applies to both parameters. First, sensitivity coefficients are used to examine process interactions and characteristics of the new optimal solution, corresponding to a decrease in R_1. A perturbation analysis is then presented to validate the sensitivity results.

6.4.1.1. Design Sensitivity

Normalized sensitivity coefficients of state and decision variables for parameter R_1 are presented in Table 6.4. The sensitivity of the reaction rate, K_1, is very high, indicating a one percent decrease in R_1 will decrease K_1 by 23.05%. This result could be obtained by differentiating equation 6.7 with respect to R_1, at the optimal digester temperature of 60 ℃. The mathematical form of the rate expression results in a large sensitivity for K_1, and is responsible for the relatively high sensitivities of some sludge treatment variables.

As indicated by sensitivities of the design variables, the liquid treatment portion of the plant is relatively unaffected by a variation in R_1. However, two changes are worth mentioning because they illustrate interactions that exist between the liquid and sludge handling subsystems.

If R_1 decreases 1% (analogously, if K_1 decreases 23.05%), underflows from both the primary and final clarifiers decrease, combining to decrease the flow rate into the gravity thickener by about 0.1%. This decrease outweighs the slight increase in sludge concentration; calculations indicate that a 1% decrease in R_1 will also reduce the optimal mass flow of solids to the thickener from 251.3 kg/hr to 231.4 kg/hr. Thus, the sludge subsystem handles a more concentrated sludge, and less of it.

Another solids/liquid interaction is indicated by the sensitivity coefficients. A decrease in R_1 produces an increase in fixed and, to a lesser extent, inert solids concentration at the head of the plant. Since the model is steady state, the increase in concentration must come from increased recycle flows; sensitivity coefficients of the gravity thickener and vacuum filter supernatant flow support this conclusion. As R_1 decreases, it becomes more expensive to stabilize solids in the digester, and the optimal design should increase recycle flows and therefore the solids loading on the liquid treatment train. This illustrates an economic tradeoff which could not be examined without a complete model including both liquid and sludge subsystems, and recycle streams from the

Table 6.4 - Normalized sensitivity coefficients of decision and state variables for R_1

Variable	S_n
Design Variables	
Primary Settler Overflow Rate	$0.0000\times10^{+1}$
Solids Retention Time in Aeration Tank	$-.2202\times10^{-1}$
Hydraulic Retention Time in Aeration Tank	0.3256×10^{-1}
Recycle Ratio of Activated Sludge	0.1485×10^{-1}
Solids Loading on Gravity Thickener	$0.2894\times10^{+1}$
Primary Digester Temperature	$0.0000\times10^{+1}$
Solids Retention Time in Digester	$-.1469\times10^{+2}$
Solids Loading on Secondary Digester	$0.6124\times10^{+1}$
Vacuum Filter Yield	$-.2036\times10^{+1}$
State Variables	
Flow into Primary Settler	0.1293×10^{-2}
Soluble BOD_5 Concentration into Primary Settler	0.2804×10^{-1}
Active Biomass Concentration into Primary Settler	$-.5116\times10^{-1}$
Degradable Solids Concentration into Primary Settler	$-.3772\times10^{-2}$
Inert Solids Concentration into Primary Settler	$-.9579\times10^{-1}$
Fixed Solids Concentration into Primary Settler	$0.3000\times10^{+0}$
Total Solids Concentration into Primary Settler	0.5449×10^{-1}
Area of Primary Settler	0.1181×10^{-2}
Solids Removal Efficiency of Primary Settler	$-.9577\times10^{-2}$
Flow Rate into Aeration Tank	0.1181×10^{-2}
Primary Settler Underflow Flow Rate	$0.1092\times10^{+0}$
Volume of Aeration Tank	0.3374×10^{-1}
Active Biomass Concentration in Aeration Tank	$-.4172\times10^{-1}$
Soluble BOD_5 Concentration out of Aeration Tank	0.2693×10^{-1}
Inert to Active Microorganism Ratio in Aeration Tank	$-.1105\times10^{+0}$
Fixed to Active Microorganism Ratio in Aeration Tank	$0.2819\times10^{+0}$
Waste sludge Ratio	0.8833×10^{-1}
Combined Recycle and Waste sludge Ratios	0.1859×10^{-1}
Active Biomass Concentration in Effluent	$-.5221\times10^{-1}$
Active Biomass Concentration in Return Sludge	$-.5843\times10^{-1}$
Area of Final Settler	0.5014×10^{-2}
Total BOD_5 removed in Aeration Tank	0.1109×10^{-1}
Diffused Air Aeration Air Flow Rate	0.1132×10^{-1}
Waste Sludge Flow Rate to Gravity Thickener	0.8951×10^{-1}
Return Sludge Total Solids Concentration	$-.6214\times10^{-2}$
Total Sludge Flow Rate into Gravity Thickener	0.9215×10^{-1}
Soluble BOD_5 Concentration of Combined Sludge	0.3344×10^{-1}
Total Solids Concentration of Combined Sludge	$-.1500\times10^{-1}$
Area of Gravity Thickener	$-.2810\times10^{+1}$

Variable	S_n
Gravity Thickener Supernatant Flow Rate	$-.1009 \times 10^{+1}$
Sludge Flow Rate into Primary Digester	$0.1924 \times 10^{+1}$
Total Solids Concentration into Primary Digester	$-.1841 \times 10^{+1}$
Fixed Solids Concentration into Primary Digester	$-.1604 \times 10^{+1}$
$Q_7 M_{a_5} M_{t_{10}} / Q_9 / M_{t_9}$	$-.4606 \times 10^{-1}$
$1000\, Q_8 M_{t_8} M_{t_{10}} / Q_9 / M_{t_9} / M_{t_1}$	$-.6100 \times 10^{-1}$
Volume of Primary Digester	$-.1277 \times 10^{+2}$
First Order Solids Stabilization Rate	$0.2305 \times 10^{+2}$
Total Heat Requirement for Digester	0.3970×10^{-2}
Total Volatile Solids into Digester	$-.1930 \times 10^{+1}$
Inert Solids Concentration in Digester Effluent	$-.9472 \times 10^{+1}$
Methane Production Rate	$0.8117 \times 10^{+0}$
Net Energy Value of Digester	$0.1236 \times 10^{+1}$
Area of Secondary Digester	$-.7422 \times 10^{+1}$
Flow Rate of Digester Supernatant	$0.0000 \times 10^{+1}$
Secondary Digester Underflow Flow Rate	$0.1924 \times 10^{+1}$
Total Solids Concentration in Digester Underflow	$-.3223 \times 10^{+1}$
Ratio of Inert to Total Solids Concentration out of Digester	$-.6249 \times 10^{+1}$
Area of Vacuum Filter	$0.3701 \times 10^{+0}$
Vacuum Filter Supernatant Flow Rate	$0.2407 \times 10^{+1}$
Filter Cake Flow Rate	$-.1666 \times 10^{+1}$
Filter Cake Solids Concentration	$0.0000 \times 10^{+1}$
Mass Fraction of Primary Sludge in Combined Sludge	$-.6511 \times 10^{-2}$
Combined Sludge Thickening Characteristic	$-.8822 \times 10^{-2}$
Combined Sludge Thickening Characteristic	$-.5247 \times 10^{-3}$
Primary Settler Underflow Solids Concentration	$-.3852 \times 10^{-1}$

sludge handling unit processes.

The sludge subsystem design variable sensitivities indicate the design of solids handling unit processes is very sensitive to a variation in R_1. Because of the lower mass flow rate of sludge into the gravity thickener, and a 2.81% increase in thickener area, the optimal thickener solids loading (L_{gt}) should decrease 2.89% with a 1% decrease in R_1. The decrease in L_{gt} produces a thicker underflow and reduces the cost of heating sludge to the digestion temperature.

The optimal solids retention time in the primary digester (θ_d) is very sensitive to a change in the rate parameter. A 1% decrease in R_1 causes a 14.7% increase in θ_d. Although costs associated with the primary digester comprise 11.7% of the total annual cost of the base optimal solution, anaerobic stabilization of solids is very important because of high dewatering and sludge disposal costs. The large increase in θ_d, partially offsetting the decrease in K_1, helps retain a high degree of solids destruction in the digester, and helps maintain methane production. The energy value of methane is credited to the total annual cost.

Again driven by the high cost of sludge dewatering and disposal, the secondary digester solids loading decreases 6.1% in response to a 1% decrease in R_1. The underflow concentration of the digested sludge increases by 3.2%, and this allows the vacuum filter area to decrease 0.37%. A decrease in the reaction rate makes the secondary digester a more cost effective unit process, because of the increased primary digester effluent solids concentration.

A perturbation analysis was performed to test the optimal changes predicted by the sensitivity coefficients. R_1 was decreased 5%; the new rate expression is presented in Figure 6.1. It is interesting that optimization in the perturbed feasible region terminated at a Kuhn-Tucker point: the only time in this study that this high confidence termination criteria was met. The decision variables for the base solution, the perturbed problem, and those predicted by sensitivity coefficients for the perturbed problem are given in Table 6.5. It is remarkable that linear estimates of change agree well with the perturbation results, given the highly nonlinear nature of the model.

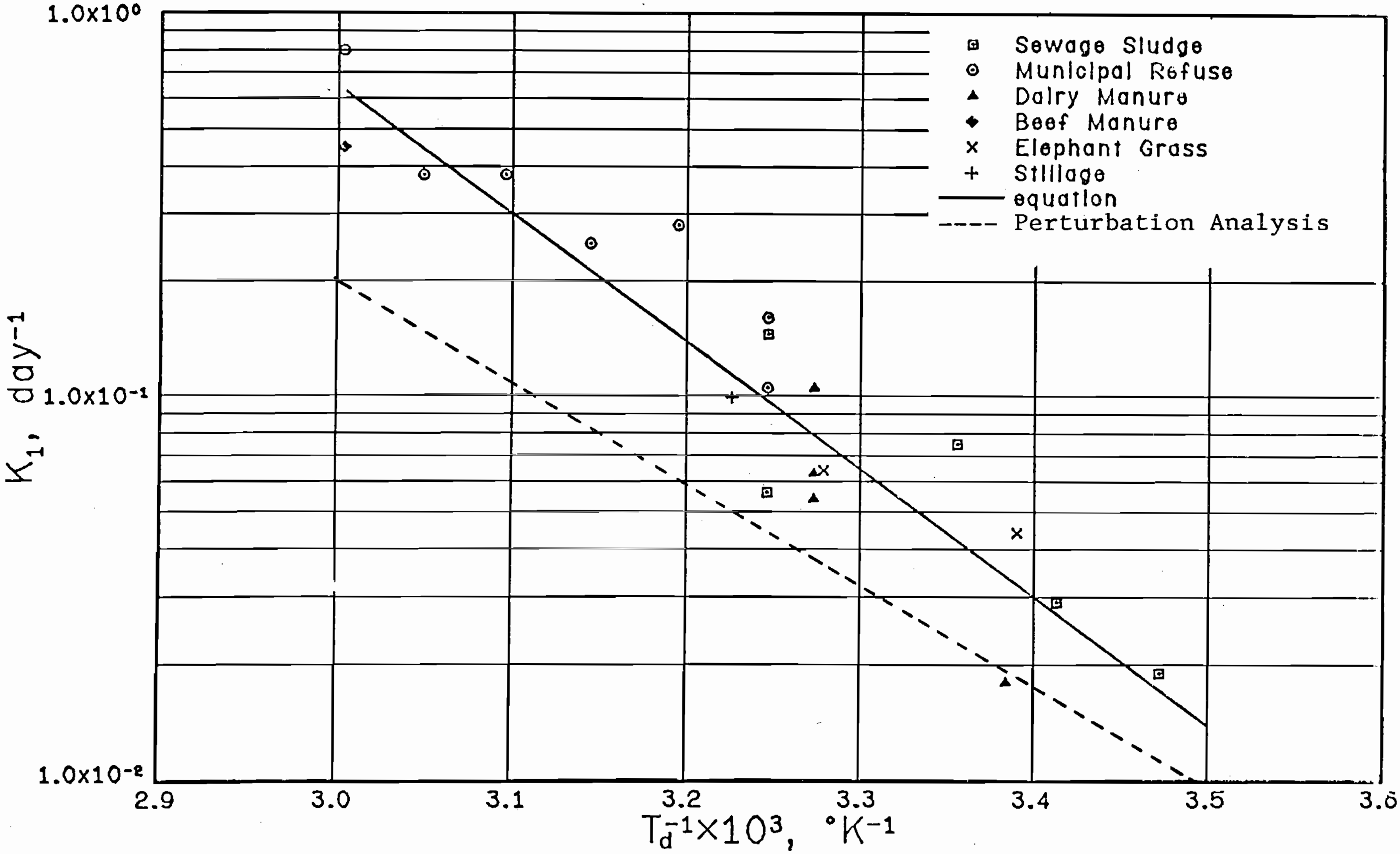

Figure 6.1 - Primary digester first order reaction rate as a function of digester temperature - perturbation of R_1

Table 6.5-Comparison of Perturbation Analysis and Estimated Solution for R_1

Variable	Solution: Base	Perturbation	Estimated
OR_p (m/hr)	6.00*	6.00*	6.00*
θ_c (days)	2.19	2.19	2.19
θ_{at} (days)	0.16	0.16	0.16
RR	0.13	0.13	0.13
L_{gt} (kg/day/m^2)	12.35	12.00*	10.56†
T_d (°C)	60.00*	60.00*	60.00*
θ_d (days)	14.62	25.63	25.36
L_d (kg/day/m^2)	40.17	30.69	27.87
L_f (kg/hr/m^2)	6.69	7.32	7.37

* Variable is at bound

† Estimation violates bound

6.4.1.2. Optimal Cost Sensitivity

Normalized sensitivity coefficients which predict the relative change in each component of the objective function are presented for the rate parameter, R_1, in Table 6.6. The column and row totals are not simply sums of the individual sensitivities in a row or column since normalized sensitivities are not meaningfull when added together. Rather, the totals represent the percent change in total annual cost for one unit process or the percent change in total annual costs of capital, operation, maintenance, materials and supply, and power.

The total objective sensitivity indicates the optimal total annual cost will increases approximately 1.5% if R_1 decreases 1.0%. The optimal objective function value of the perturbed problem is greater than the objective function estimated using the sensitivity coefficients; $546,431 versus $536,564. The difference is due to not considering second order (and higher order) effects of a parameter change and to the gravity thickener solids loading being at a bound at the perturbed optimum. The total capital cost is most sensitive to a change in R_1 because the gravity thickener and primary and secondary digesters increase

Table 6.6 - Normalized objective sensitivity coefficients for R_1

	CAPITAL	OPERATION	MAINTENANCE	MATERIAL	POWER	TOTAL
PRIMARY SETTLING TANK	$0.90909\text{x}10^{-3}$	$0.35419\text{x}10^{-3}$	$0.16529\text{x}10^{-3}$	$0.89729\text{x}10^{-3}$		$0.72458\text{x}10^{-3}$
PRIMARY SLUDGE PUMPING	$0.57849\text{x}10^{-1}$	$0.44751\text{x}10^{-1}$	$0.46934\text{x}10^{-1}$	$0.69856\text{x}10^{-1}$	$0.10915\text{x}10^{+0}$	$0.53150\text{x}10^{-1}$
AERATION TANK	$0.23956\text{x}10^{-1}$					$0.23956\text{x}10^{-1}$
DIFFUSED AIR AERATION	$0.74698\text{x}10^{-2}$	$0.54325\text{x}10^{-2}$	$0.62248\text{x}10^{-2}$			$0.68939\text{x}10^{-2}$
SECONDARY SETTLING TANK	$0.38607\text{x}10^{-2}$	$0.30083\text{x}10^{-2}$	$0.30083\text{x}10^{-2}$	$0.38106\text{x}10^{-2}$		$0.36293\text{x}10^{-2}$
RECYCLE SLUDGE PUMPING	$0.10478\text{x}10^{-1}$	$0.19770\text{x}10^{-1}$	$0.19770\text{x}10^{-1}$	$0.10280\text{x}10^{-1}$	$0.19770\text{x}10^{-1}$	$0.13143\text{x}10^{-1}$
GRAVITY THICKENER	$-.21637\text{x}10^{+1}$	$-.16860\text{x}10^{+1}$	$-.16860\text{x}10^{+1}$	$-.21356\text{x}10^{+1}$		$-.20285\text{x}10^{+1}$
PRIMARY ANAEROBIC DIGESTER	$-.75322\text{x}10^{+1}$	$-.25533\text{x}10^{+1}$	$-.26809\text{x}10^{+1}$	$-.47236\text{x}10^{+2}$		$-.62654\text{x}10^{+1}$
SECONDARY ANAEROBIC DIGESTER	$-.43789\text{x}10^{+1}$	$-.14844\text{x}10^{+1}$	$-.15586\text{x}10^{+1}$	$-.27461\text{x}10^{+1}$		$-.34175\text{x}10^{+1}$
VACUUM FILTER	$0.26278\text{x}10^{+0}$	$-.96649\text{x}10^{+0}$	$-.79985\text{x}10^{+0}$	$-.13339\text{x}10^{+1}$		$-.57469\text{x}10^{+0}$
RECYCLE STREAM PUMPING	$0.92172\text{x}10^{-1}$	$0.17391\text{x}10^{+0}$	$0.17391\text{x}10^{+0}$	$0.00000\text{x}10^{+1}$	$0.17391\text{x}10^{+0}$	$0.79140\text{x}10^{-1}$
FINAL SLUDGE DISPOSAL	$-.12837\text{x}10^{+1}$	$-.11114\text{x}10^{+1}$				$-.11464\text{x}10^{+1}$
TOTALS FOR C/O/M/MAT/P	$-.14214\text{x}10^{+1}$	$-.86656\text{x}10^{+0}$	$-.71508\text{x}10^{+0}$	$-.16090\text{x}10^{+1}$	$0.28574\text{x}10^{-1}$	

NET ENERGY FROM METHANE = $0.12355\text{x}10^{+1}$

TOTAL OBJECTIVE SENSITIVITY = $-.14449\text{x}10^{+1}$

in size and the amount of land required for sludge disposal of the filter cake increases. The capital cost of the primary digester has the largest sensitivity coefficient.

6.4.1.3. Summary

The liquid treatment subsystem design is relatively insensitive to changes in the rate parameters R_1 and R_2. A decrease in parameter R_1 increases the cost of anaerobic sludge stabilization. To remain optimal with respect to annual cost, the mass flow of sludge to the solids handling subsystem should be reduced. A larger gravity thickener should be used to reduce heat requirements in the digester, and the solids retention time in the digester should be increased to maintain a high degree of solids destruction. The secondary digester becomes more cost effective for thickening the digested sludge to reduce dewatering and sludge disposal costs. The percent increase in total annual cost should be greater than the percent reduction in R_1.

Given the uncertainty in the digester rate expression and values of the coefficients, and the sensitivity of the sludge subsystem design and optimal annual cost, R_1 and R_2 are certainly important parameters in the wastewater treatment plant model.

6.4.2. Variation in Activated Sludge Thickening Characteristics

Thickening of the activated sludge in the final settler is assumed to obey equation 3.2, and the thickening parameters for the activated sludge are denoted a_w and n_w. Thickening of the activated sludge is very important; to maintain biological removal of oxygen demanding material the active microorganisms must be thickened, and then a portion returned to the aeration basin. If the thickening characteristics are very poor, the quantity of solids recycled will be reduced as active microorganisms are lost in the plant effluent, potentially violating both *TSS* and BOD_5 effluent requirements. Thus, it is important to investigate the sensitivity of the optimal design and annual cost to changes in activated sludge thickening characteristics. This sensitivity analysis can illustrate how to design a plant containing a safety factor to protect against sludges with poor settling qualities. In practice, the settling properties of activated sludge may vary with influent waste characteristics or changing plant operating conditions.

The sensitivity analysis presented considers only the effect of an increase in n_w. This simulates a sludge with poorer settling qualities due to an increase in the inhibitory effect of particle interactions. The sensitivity coefficients are examined initially to illustrate the interactions among unit processes and to examine how the optimal solution changes when n_w increases. A perturbation analysis is then presented to confirm the sensitivity coefficient results.

6.4.2.1. Design Sensitivity

Sensitivity coefficients of design and state variables are presented in Table 6.7. Excluding the primary clarifier design, the coefficients predict that a decrease in n_w affects the design of both liquid and solids handling unit processes significantly.

The solids retention time in the aeration tank (θ_c) is not appreciably affected by a small increase in n_w. The insensitivity of θ_c to a change in n_w indicates the importance of maintaining sufficient biological BOD_5 reduction so effluent requirements are met. For a 1% increase in n_w, θ_{at} and the sludge recycle ratio (RR) would increase 1.08% and 1.91%, respectively. This is necessary to maintain θ_c since the poorer thickening quality of the activated sludge decreases the final clarifier underflow concentration by 2.86%.

The final clarifier efficiency, as predicted by the Chapman model (equation 3.3), is not directly affected by a change in n_w. The mass balance of solids in the activated sludge system must be satisfied, but since the clarifier efficiency is insensitive to n_w, the cost effective way to balance the solids is to increase the underflow rate, not the final clarifier area (thereby increasing underflow concentration). This decision is optimal even though increased flows are sent to the solids handling subsystem.

The sludge solids concentration entering the gravity thickener decreases 2.45% in response to a 1% increase in n_w. The flow rate of sludge into the solids handling system increases by about the same percentage. The gravity thickener solids loading should decrease 2.13% to minimize the increase in

Table 6.7 - Normalized sensitivity coefficients of decision and state variables for n_w

Variable	S_n
Design Variables	
Primary Settler Overflow Rate	$0.0000\times10^{+1}$
Solids Retention Time in Aeration Tank	$-.1842\times10^{-1}$
Hydraulic Retention Time in Aeration Tank	$0.1081\times10^{+1}$
Recycle Ratio of Activated Sludge	$0.1913\times10^{+1}$
Solids Loading on Gravity Thickener	$-.2127\times10^{+1}$
Primary Digester Temperature	$0.0000\times10^{+1}$
Solids Retention Time in Digester	$-.1017\times10^{+1}$
Solids Loading on Secondary Digester	$0.2636\times10^{+1}$
Vacuum Filter Yield	$-.8766\times10^{+0}$
State Variables	
Flow into Primary Settler	0.1968×10^{-1}
Soluble BOD_5 Concentration into Primary Settler	0.7923×10^{-2}
Active Biomass Concentration into Primary Settler	$0.1221\times10^{+0}$
Degradable Solids Concentration into Primary Settler	$-.1281\times10^{-1}$
Inert Solids Concentration into Primary Settler	0.5438×10^{-1}
Fixed Solids Concentration into Primary Settler	$0.1274\times10^{+0}$
Total Solids Concentration into Primary Settler	0.4270×10^{-1}
Area of Primary Settler	0.1959×10^{-1}
Solids Removal Efficiency of Primary Settler	$-.7505\times10^{-2}$
Flow Rate into Aeration Tank	0.1959×10^{-1}
Primary Settler Underflow Flow Rate	$0.1042\times10^{+0}$
Volume of Aeration Tank	$0.1101\times10^{+1}$
Active Biomass Concentration in Aeration Tank	$-.1104\times10^{+1}$
Soluble BOD_5 Concentration out of Aeration Tank	0.2253×10^{-1}
Inert to Active Microorganism Ratio in Aeration Tank	0.4181×10^{-1}
Fixed to Active Microorganism Ratio in Aeration Tank	$0.1173\times10^{+0}$
Waste sludge Ratio	$0.2893\times10^{+1}$
Combined Recycle and Waste sludge Ratios	$0.1963\times10^{+1}$
Active Biomass Concentration in Effluent	$-.4368\times10^{-1}$
Active Biomass Concentration in Return Sludge	$-.2867\times10^{+1}$
Area of Final Settler	0.1344×10^{-1}
Total BOD_5 removed in Aeration Tank	$-.5851\times10^{-2}$
Diffused Air Aeration Air Flow Rate	0.1295×10^{-1}
Waste Sludge Flow Rate to Gravity Thickener	$0.2913\times10^{+1}$
Return Sludge Total Solids Concentration	$-.2824\times10^{+1}$
Total Sludge Flow Rate into Gravity Thickener	$0.2536\times10^{+1}$
Soluble BOD_5 Concentration of Combined Sludge	$-.8453\times10^{+0}$
Total Solids Concentration of Combined Sludge	$-.2454\times10^{+1}$
Area of Gravity Thickener	$0.2191\times10^{+1}$

Variable	S_n
Gravity Thickener Supernatant Flow Rate	$0.3039 \times 10^{+1}$
Sludge Flow Rate into Primary Digester	$0.1697 \times 10^{+1}$
Total Solids Concentration into Primary Digester	$-.1633 \times 10^{+1}$
Fixed Solids Concentration into Primary Digester	$-.1554 \times 10^{+1}$
$Q_7 M_{a_5} M_{t_{10}} / Q_9 / M_{t_9}$	$-.3650 \times 10^{-1}$
$1000\, Q_8 M_{t_8} M_{t_{10}} / Q_9 / M_{t_9} / M_{t_1}$	$-.5031 \times 10^{-1}$
Volume of Primary Digester	$0.6799 \times 10^{+0}$
First Order Solids Stabilization Rate	$0.0000 \times 10^{+1}$
Total Heat Requirement for Digester	$0.1564 \times 10^{+1}$
Total Volatile Solids into Digester	$-.1663 \times 10^{+1}$
Inert Solids Concentration in Digester Effluent	$-.7448 \times 10^{+0}$
Methane Production Rate	$-.6416 \times 10^{-1}$
Net Energy Value of Digester	$-.9187 \times 10^{+0}$
Area of Secondary Digester	$-.2326 \times 10^{+1}$
Flow Rate of Digester Supernatant	$0.0000 \times 10^{+0}$
Secondary Digester Underflow Flow Rate	$0.1697 \times 10^{+1}$
Total Solids Concentration in Digester Underflow	$-.1387 \times 10^{+1}$
Ratio of Inert to Total Solids Concentration out of Digester	$0.6424 \times 10^{+0}$
Area of Vacuum Filter	$0.1028 \times 10^{+1}$
Vacuum Filter Supernatant Flow Rate	$0.1905 \times 10^{+1}$
Filter Cake Flow Rate	$0.1514 \times 10^{+0}$
Filter Cake Solids Concentration	$0.0000 \times 10^{+1}$
Mass Fraction of Primary Sludge in Combined Sludge	$-.7605 \times 10^{-2}$
Combined Sludge Thickening Characteristic	$-.1030 \times 10^{-1}$
Combined Sludge Thickening Characteristic	$0.5137 \times 10^{+0}$
Primary Settler Underflow Solids Concentration	$-.3018 \times 10^{-1}$

heating costs required to bring the sludge to the digester temperature. Solids destruction in the primary digester is not as economical when n_w increases; this is evident by the decrease in θ_d. The low sludge solids concentration, which makes the primary digester less cost effective, also decreases the importance of thickening the digested sludge in the secondary digester. Thus, L_d should increase 2.63% in response to a 1% increase in n_w. Because of high sludge disposal costs, the mass flow rate of the filter cake should be minimized. When n_w increases 1%, it is optimal for more dewatering of the sludge to be done mechanically, as shown by the 1.02% increase in vacuum filter area.

The nominal value of n_w was perturbed 12.6% from 2.3747 to 2.6747. An optimal solution was obtained for the new feasible region. It is presented in Table 6.8, along with the base optimum and the decision vector predicted by the sensitivity coefficients.

Table 6.8-Comparison of Perturbation Analysis and Estimated Solution for n_w

	Solution		
Variable	Base	Perturbation	Estimated
OR_p (m/hr)	6.00*	5.92	6.00*
θ_c (days)	2.19	2.18	2.19
θ_{at} (days)	0.16	0.18	0.18
RR	0.13	0.15	0.16
L_{gt} (kg/day/m^2)	12.35	12.00*	9.03†
T_d (℃)	60.00*	60.00*	60.00*
θ_d (days)	14.62	11.78	12.74
L_d (kg/day/m^2)	40.17	48.00*	53.54†
L_f (kg/hr/m^2)	6.69	6.31	5.95

* Variable is at bound

† Estimation violates bound

As discussed previously, the sensitivity coefficients always predict zero change for a variable at bound at the optimum. The perturbation analysis indicates that this prediction is not always correct. The primary clarifier was released from its upper bound in the perturbed optimal solution. As a result, the perturbation analysis yielded a slightly lower value of θ_c than the value of θ_c estimated using the sensitivity coefficients, primarily due to a lower recycle flow. Although not shown in Table 6.8, the supernatant flow from the secondary digester increased from its lower bound of essentially zero to 1.18 m^3/hr. The increase in the supernatant flow allowed a thicker underflow solids concentration and, therefore, a smaller vacuum filter than predicted. Other factors contributing to some differences between estimated and actual values of decision variables include the gravity thickener and secondary digester reaching their bounds and, of course, second order effects not taken into account by the sensitivity coefficients.

6.4.2.2. Optimal Cost Sensitivity

Normalized objective sensitivities for a change in n_w are presented in Table 6.9. The total objective sensitivity coefficient can be used to estimate a new objective value for the perturbed problem of \$523,326; this value agrees closely with the perturbation result of \$524,870. The costs of the aeration tank, recycle sludge pumping, and all solids handling unit processes are relatively sensitive to a change in n_w. The capital costs are most sensitive; percentage changes in total operation, maintenance, material and supply, and power are roughly half those of the capital expenditures. Note the primary digester capital cost increases even though θ_d decreases. This cost increase is due to increased flows through the sludge treatment train, mainly from the final clarifier. Again, the thin digested sludge makes it less economical to build the secondary digester, but the relatively low sensitivity of sludge disposal costs indicates that the vacuum filter has increased in capacity to dewater the sludge adequately prior to landfilling.

6.4.2.3. Summary

Almost every unit process is sensitive to a change in activated sludge settling characteristics, as represented by a change in n_w. An optimal solution corresponding to an increase in n_w has larger primary and aeration basins, and larger recycle flows of the thinner waste activated sludge. The thinner sludge

Table 6.9 - Normalized objective sensitivity coefficients for n_w

	CAPITAL	OPERATION	MAINTENANCE	MATERIAL	POWER	TOTAL
PRIMARY SETTLING TANK	$0.15087x10^{-1}$	$0.58784x10^{-2}$	$0.27432x10^{-2}$	$0.14892x10^{-1}$		$0.12025x10^{-1}$
PRIMARY SLUDGE PUMPING	$0.55228x10^{-1}$	$0.42723x10^{-1}$	$0.44807x10^{-1}$	$0.66690x10^{-1}$	$0.10420x10^{+0}$	$0.50742x10^{-1}$
AERATION TANK	$0.78146x10^{+0}$					$0.78146x10^{+0}$
DIFFUSED AIR AERATION	$0.85470x10^{-2}$	$0.62160x10^{-2}$	$0.71225x10^{-2}$			$0.78881x10^{-2}$
SECONDARY SETTLING TANK	$0.10348x10^{-1}$	$0.80637x10^{-2}$	$0.80637x10^{-2}$	$0.10214x10^{-1}$		$0.97281x10^{-2}$
RECYCLE SLUDGE PUMPING	$0.10505x10^{+1}$	$0.19822x10^{+1}$	$0.19822x10^{+1}$	$0.10307x10^{+1}$	$0.19822x10^{+1}$	$0.13178x10^{+1}$
GRAVITY THICKENER	$0.16869x10^{+1}$	$0.13144x10^{+1}$	$0.13144x10^{+1}$	$0.16650x10^{+1}$		$0.15815x10^{+1}$
PRIMARY ANAEROBIC DIGESTER	$0.40112x10^{+0}$	$0.13597x10^{+0}$	$0.14277x10^{+0}$	$0.25155x10^{+0}$		$0.33366x10^{+0}$
SECONDARY ANAEROBIC DIGESTER	$-.13723x10^{+1}$	$-.46520x10^{+0}$	$-.48846x10^{+0}$	$-.86063x10^{+0}$		$-.10710x10^{+1}$
VACUUM FILTER	$0.72987x10^{+0}$	$0.87819x10^{-1}$	$0.72678x10^{-1}$	$0.12121x10^{+0}$		$0.36240x10^{+0}$
RECYCLE STREAM PUMPING	$0.14026x10^{+1}$	$0.26464x10^{+1}$	$0.26464x10^{+1}$	$0.00000x10^{+1}$	$0.26464x10^{+1}$	$0.12043x10^{+1}$
FINAL SLUDGE DISPOSAL	$0.11664x10^{+0}$	$0.10099x10^{+0}$				$0.10416x10^{+0}$
TOTALS FOR C/O/M/MAT/P	$0.34143x10^{+0}$	$0.11520x10^{+0}$	$0.12513x10^{+0}$	$0.15311x10^{+0}$	$0.20033x10^{+1}$	

NET ENERGY FROM METHANE = $-.91868x10^{+0}$

TOTAL OBJECTIVE SENSITIVITY = $0.36310x10^{+0}$

increases costs of sludge handling. A larger gravity thickener will increase the underflow solids concentration and minimize the heating penalty in the primary digester. The secondary digester becomes inefficient unless the digested sludge is concentrated, and the model predicts its elimination from the process train when n_w is high. More burden is placed upon the vacuum filter to dewater the thinner sludge and minimize the penalty of increased sludge disposal costs.

Given the sensitivity of unit processes and optimal cost to a change in n_w, and that n_w may fluctuate with process control changes and waste characteristics, the thickening characteristic is an important model parameter.

6.4.3. Variation in Aeration Basin Oxygen Concentration

The dissolved oxygen concentration in the aeration basin (DO_{at}) is a parameter of the model. The nominal value of DO_{at} is 1.5 mg/l. Oxygen utilization is computed using the Lawrence-McCarty model [1970], and the required air flow rate is then a function of the substrate utilization, the efficiency of the transfer process, the temperature of the water, and the DO_{at} desired. When nitrogenous BOD_5 removal is required, DO_{at} must be higher to allow growth of nitrifying microorganisms that convert NH_3 to nitrates and nitrites. It is desireable, then, to examine the optimal change in design associated with a higher DO_{at} level. The change in optimal annual cost should indicate part of the cost penalty for the higher DO_{at} level when nitrification is desired. The entire cost penalty would be greater since a longer θ_c is also required.

Design sensitivity predicted by the sensitivity coefficients is first examined. A perturbation analysis is then presented to illustrate the affect of increasing DO_{at} from 1.5 to 4.0 mg/l.

6.4.3.1. Design Sensitivity

Sensitivity coefficients for state and decision variables are presented in Table 6.10 The optimal design is completely insensitive to a change in the DO_{at} level. The only model variable sensitive at all to a small change in DO_{at} is Q_a, the air flow rate. The insensitivity of the model is counter-intuitive, and an

Table 6.10 - Normalized sensitivity coefficients of decision and state variables for DO_{at}

Variable	S_n
Design Variables	
Primary Settler Overflow Rate	$0.0000 \times 10^{+1}$
Solids Retention Time in Aeration Tank	0.5104×10^{-5}
Hydraulic Retention Time in Aeration Tank	0.8897×10^{-4}
Recycle Ratio of Activated Sludge	$-.1856 \times 10^{-3}$
Solids Loading on Gravity Thickener	$-.7477 \times 10^{-3}$
Primary Digester Temperature	$0.0000 \times 10^{+1}$
Solids Retention Time in Digester	0.2668×10^{-3}
Solids Loading on Secondary Digester	$-.7782 \times 10^{-3}$
Vacuum Filter Yield	0.2588×10^{-3}
State Variables	
Flow into Primary Settler	$-.5394 \times 10^{-6}$
Soluble BOD_5 Concentration into Primary Settler	$-.6288 \times 10^{-5}$
Active Biomass Concentration into Primary Settler	0.9729×10^{-5}
Degradable Solids Concentration into Primary Settler	0.9984×10^{-6}
Inert Solids Concentration into Primary Settler	$-.1520 \times 10^{-4}$
Fixed Solids Concentration into Primary Settler	$-.3756 \times 10^{-4}$
Total Solids Concentration into Primary Settler	$-.1259 \times 10^{-4}$
Area of Primary Settler	$-.5133 \times 10^{-6}$
Solids Removal Efficiency of Primary Settler	0.2212×10^{-5}
Flow Rate into Aeration Tank	$-.5133 \times 10^{-6}$
Primary Settler Underflow Flow Rate	$-.2546 \times 10^{-4}$
Volume of Aeration Tank	0.8846×10^{-4}
Active Biomass Concentration in Aeration Tank	$-.8664 \times 10^{-4}$
Soluble BOD_5 Concentration out of Aeration Tank	$-.6241 \times 10^{-5}$
Inert to Active Microorganism Ratio in Aeration Tank	$-.1053 \times 10^{-4}$
Fixed to Active Microorganism Ratio in Aeration Tank	$-.3343 \times 10^{-4}$
Waste sludge Ratio	$-.7756 \times 10^{-4}$
Combined Recycle and Waste sludge Ratios	$-.1801 \times 10^{-3}$
Active Biomass Concentration in Effluent	0.1210×10^{-4}
Active Biomass Concentration in Return Sludge	0.7059×10^{-4}
Area of Final Settler	$-.4171 \times 10^{-4}$
Total BOD_5 removed in Aeration Tank	$-.2367 \times 10^{-5}$
Diffused Air Aeration Air Flow Rate	$0.2080 \times 10^{+0}$
Waste Sludge Flow Rate to Gravity Thickener	$-.7808 \times 10^{-4}$
Return Sludge Total Solids Concentration	0.5849×10^{-4}
Total Sludge Flow Rate into Gravity Thickener	$-.7101 \times 10^{-4}$
Soluble BOD_5 Concentration of Combined Sludge	0.9879×10^{-5}
Total Solids Concentration of Combined Sludge	0.5290×10^{-4}
Area of Gravity Thickener	0.7284×10^{-3}

Variable	S_n
Gravity Thickener Supernatant Flow Rate	0.1837×10^{-3}
Sludge Flow Rate into Primary Digester	$-.4950 \times 10^{-3}$
Total Solids Concentration into Primary Digester	0.4757×10^{-3}
Fixed Solids Concentration into Primary Digester	0.4526×10^{-3}
$Q_7 M_{a_5} M_{t_{10}} / Q_9 / M_{t_9}$	0.1063×10^{-4}
$1000 Q_8 M_{t_8} M_{t_{10}} / Q_9 / M_{t_9} / M_{t_1}$	0.1415×10^{-4}
Volume of Primary Digester	$-.2281 \times 10^{-3}$
First Order Solids Stabilization Rate	$0.0000 \times 10^{+1}$
Total Heat Requirement for Digester	$-.4601 \times 10^{-3}$
Total Volatile Solids into Digester	0.4844×10^{-3}
Inert Solids Concentration in Digester Effluent	0.2436×10^{-3}
Methane Production Rate	0.1503×10^{-4}
Net Energy Value of Digester	0.2466×10^{-3}
Area of Secondary Digester	0.6928×10^{-3}
Flow Rate of Digester Supernatant	$0.0000 \times 10^{+1}$
Secondary Digester Underflow Flow Rate	$-.4950 \times 10^{-3}$
Total Solids Concentration in Digester Underflow	0.4096×10^{-3}
Ratio of Inert to Total Solids Concentration out of Digester	$-.1659 \times 10^{-3}$
Area of Vacuum Filter	$-.2974 \times 10^{-3}$
Vacuum Filter Supernatant Flow Rate	$-.5562 \times 10^{-3}$
Filter Cake Flow Rate	$-.3864 \times 10^{-4}$
Filter Cake Solids Concentration	$0.0000 \times 10^{+1}$
Mass Fraction of Primary Sludge in Combined Sludge	0.1560×10^{-5}
Combined Sludge Thickening Characteristic	0.2113×10^{-5}
Combined Sludge Thickening Characteristic	0.1257×10^{-6}
Primary Settler Underflow Solids Concentration	0.8898×10^{-5}

investigation of the model equations is necessary to justify the results of the sensitivity analysis.

It is reasonable to expect sludge age to decrease in response to an increase in DO_{at}. A decrease in sludge age reduces the oxygen requirement since more BOD_5 is converted to cell material and wasted from the activated sludge subsystem. However, Table 6.2 indicates that θ_c is relatively insensitive to almost every parameter in the model, except some kinetic coefficients and the effluent standards. This insensitivity is a product of the final clarifier model used to predict solids removal efficiency and of the assumption that the values of θ_c and other activated sludge parameters are independent of DO_{at}. Since θ_c is so insensitive, so is the design insensitive to changes in DO_{at} since the DO_{at} does not interact with any other model variable.

The DO_{at} level was increased from 1.5 to 4.0 mg/l, and the optimal solution of the perturbed problem obtained. The decision variables of the base optimum, the perturbed optimum, and those estimated using the sensitivity coefficients are presented in Table 6.11. The perturbation analysis and sensitivity coefficents both indicate that the optimal design is insensitive to a reasonable change in DO_{at}. The insensitivity may indicate a deficiency in the process model. If higher DO_{at} levels are shown to affect performance in practice, then this process interaction should be evaluated and included in future models of this type.

6.4.3.2. Optimal Cost Sensitivity

The optimal solution of the perturbed problem has an objective value of \$533,460, a 6.6% increase over the base optimal solution. The objective sensitivity coefficient equals 0.0262, and can be used to estimate a total annual cost of \$523,939. Since the design sensitivities compare well, the difference between the annual cost obtained by the perturbation anaysis and the annual cost obtained using the sensitivity coefficients is probably due to nonlinearities of the cost function for diffused air aeration. Though the design is insensitive to a change in DO_{at}, consideration of the DO_{at} level is important because of its potential influence on the total annual cost.

Table 6.11-Comparison of Perturbation Analysis and Estimated solution for DO_{at}

	Solution		
Variable	Base	Perturbation	Estimated
OR_p (m/hr)	6.00*	6.00*	6.00*
θ_c (days)	2.19	2.19	2.19
θ_{at} (days)	0.16	0.16	0.16
RR	0.13	0.13	0.13
L_{gt} (kg/day/m^2)	12.35	12.34	12.33
T_d (℃)	60.00*	60.00*	60.00*
θ_d (days)	14.62	14.49	14.63
L_d (kg/day/m^2)	40.17	40.00	40.12
L_f (kg/hr/m^2)	6.69	6.70	6.70

* Variable is at bound

6.4.3.3. Summary

The optimal design is insensitive to changes in DO_{at} level. This is due primarily to insensitivity of θ_c. If there is a real process interaction associated with different DO_{at} levels, then it is not included in the model. Although the design is insensitive, total annual cost may increase significantly for reasonable changes in DO_{at} level, due to an increase in Q_a.

6.4.4. Variation in Influent Wastewater Flow

Design of wastewater treatment plants usually assumes steady-state influent wastewater flow. In reality, the influent flow is not steady, but usually contains diurnal fluctuations. When the treatment plant is designed, an estimate of the future influent characteristics must be specified based on forcasting of the areas needs. Due to uncertainty and the non-steady nature of the input, this estimate is usually conservative. A sensitivity analysis is useful for determining the cost penalty associated with applying safety factors to the influent flow rate.

Sensitivity coefficients are first examined to identify characteristics of the optimal solution when influent flow rate (Q_0) increases. A perturbation analysis is then presented to examine the effect of increasing Q_0 from 1500 to 2500 m^3/hr. Results of the two sensitivity analyses are then compared to confirm optimal trends predicted by the sensitivity coefficients. Finally, detailed cost sensitivities are presented to aid in understanding the driving forces behind the design interactions.

6.4.4.1. Design Sensitivity

Sensitivity coefficients for decision and state variables are presented in Table 6.12 In the liquid treatment train, a 1% increase in Q_0 causes approximately a 1% increase in the unit sizes. Therefore, the sensitivities of the liquid handling design variables are very low. An increase in flow to the aeration basin, for instance, is accompanyed by a proportional increase in the size of the aeration tank; thus the θ_{at} does not deviate from the base optimal value. At first this result may seem intuitive. However, considering the nonlinearity of the cost functions, it is really quite surprising that the optimal values of the liquid subsystem decision variables are so robust with respect to a change in influent flow rate.

Analysis of the sensitivity coefficients of sludge treatment decision variables leads to a different result. For a higher influent flowrate, the gravity thickener becomes even more cost effective at thickening the combined sludge before it is heated in the digester. The savings in heating costs resulting from an increase in the thickener underflow solids concentration increases at a faster rate, with respect to Q_0, than does the cost of building and operating the larger thickener. The sensitivity coefficients can be used to estimate that the solids retention time in the primary digester (θ_d) should increase with an increase in influent flow, and the secondary digester becomes a more cost effective unit process for thickening the digested sludge prior to dewatering and sludge disposal. The above design changes occur because of the economies of scale in the unit process cost functions. The tradeoff which involves θ_d is increased energy value from methane and decreased sludge disposal costs versus increased capital, operation, and maintenance costs of the digester. The vacuum filter is the only sludge treatment unit process for which the fractional increase in size is less than the fractional increase in influent flow rate. A tradeoff between the

Table 6.12 - Normalized sensitivity coefficients of decision and state variables for Q_0

Variable	S_n
Design Variables	
Primary Settler Overflow Rate	$0.0000\times10^{+1}$
Solids Retention Time in Aeration Tank	0.7977×10^{-3}
Hydraulic Retention Time in Aeration Tank	0.9028×10^{-2}
Recycle Ratio of Activated Sludge	$-.1982\times10^{-1}$
Solids Loading on Gravity Thickener	$-.1204\times10^{+0}$
Primary Digester Temperature	$0.0000\times10^{+1}$
Solids Retention Time in Digester	$0.1358\times10^{+0}$
Solids Loading on Secondary Digester	$-.9458\times10^{-1}$
Vacuum Filter Yield	0.3145×10^{-1}
State Variables	
Flow into Primary Settler	$0.9999\times10^{+0}$
Soluble BOD_5 Concentration into Primary Settler	$-.1004\times10^{-2}$
Active Biomass Concentration into Primary Settler	0.1749×10^{-2}
Degradable Solids Concentration into Primary Settler	0.1426×10^{-3}
Inert Solids Concentration into Primary Settler	$-.3974\times10^{-2}$
Fixed Solids Concentration into Primary Settler	$-.4521\times10^{-2}$
Total Solids Concentration into Primary Settler	$-.1971\times10^{-2}$
Area of Primary Settler	$0.9999\times10^{+0}$
Solids Removal Efficiency of Primary Settler	0.3465×10^{-3}
Flow Rate into Aeration Tank	$0.9999\times10^{+0}$
Primary Settler Underflow Flow Rate	$0.9960\times10^{+0}$
Volume of Aeration Tank	$0.1009\times10^{+1}$
Active Biomass Concentration in Aeration Tank	$-.8685\times10^{-2}$
Soluble BOD_5 Concentration out of Aeration Tank	$-.9754\times10^{-3}$
Inert to Active Microorganism Ratio in Aeration Tank	$-.3181\times10^{-2}$
Fixed to Active Microorganism Ratio in Aeration Tank	$-.3869\times10^{-2}$
Waste sludge Ratio	$-.9247\times10^{-2}$
Combined Recycle and Waste sludge Ratios	$-.1928\times10^{-1}$
Active Biomass Concentration in Effluent	0.1891×10^{-2}
Active Biomass Concentration in Return Sludge	0.8156×10^{-2}
Area of Final Settler	$0.9955\times10^{+0}$
Total BOD_5 removed in Aeration Tank	$-.3904\times10^{-3}$
Diffused Air Aeration Air Flow Rate	$0.9996\times10^{+0}$
Waste Sludge Flow Rate to Gravity Thickener	$0.9907\times10^{+0}$
Return Sludge Total Solids Concentration	0.6264×10^{-2}
Total Sludge Flow Rate into Gravity Thickener	$0.9914\times10^{+0}$
Soluble BOD_5 Concentration of Combined Sludge	0.6504×10^{-3}
Total Solids Concentration of Combined Sludge	0.5776×10^{-2}
Area of Gravity Thickener	$0.1117\times10^{+1}$

Variable	S_n
Gravity Thickener Supernatant Flow Rate	$0.1034 \times 10^{+1}$
Sludge Flow Rate into Primary Digester	$0.9204 \times 10^{+0}$
Total Solids Concentration into Primary Digester	0.7660×10^{-1}
Fixed Solids Concentration into Primary Digester	0.7435×10^{-1}
$Q_7 M_{a_5} M_{t_{10}} / Q_9 / M_{t_9}$	0.1663×10^{-2}
$1000 Q_8 M_{t_8} M_{t_{10}} / Q_9 / M_{t_9} / M_{t_1}$	0.2213×10^{-2}
Volume of Primary Digester	$0.1056 \times 10^{+1}$
First Order Solids Stabilization Rate	$0.0000 \times 10^{+1}$
Total Heat Requirement for Digester	$0.9381 \times 10^{+0}$
Total Volatile Solids into Digester	0.7745×10^{-1}
Inert Solids Concentration in Digester Effluent	$-.4504 \times 10^{-1}$
Methane Production Rate	$0.1011 \times 10^{+1}$
Net Energy Value of Digester	$0.1049 \times 10^{+1}$
Area of Secondary Digester	$0.1065 \times 10^{+1}$
Flow Rate of Digester Supernatant	$0.0000 \times 10^{+1}$
Secondary Digester Underflow Flow Rate	$0.9204 \times 10^{+0}$
Total Solids Concentration in Digester Underflow	$0.4978 \times 10^{+1}$
Ratio of Inert to Total Solids Concentration out of Digester	$-.9481 \times 10^{-1}$
Area of Vacuum Filter	$0.9444 \times 10^{+0}$
Vacuum Filter Supernatant Flow Rate	$0.9129 \times 10^{+0}$
Filter Cake Flow Rate	$0.9758 \times 10^{+0}$
Filter Cake Solids Concentration	$0.0000 \times 10^{+0}$
Mass Fraction of Primary Sludge in Combined Sludge	0.2419×10^{-3}
Combined Sludge Thickening Characteristic	0.3277×10^{-3}
Combined Sludge Thickening Characteristic	0.1949×10^{-4}
Primary Settler Underflow Solids Concentration	0.1393×10^{-2}

costs associated with anaerobic digestion and gravity thickening and the costs associated with dewatering and land disposal of the sludge is illustrated by the sensitivity analysis. Since the latter costs are so great, they exert an additional driving force to thicken and stabilize the sludge solids. The increased influent flow rate leads to a decrease in the percentage of total annual costs attributed to dewatering and disposal of the filter cake.

The results of a perturbation analysis, in which the influent wastewater flow was increased from 1500 to 2500 m^3/hr, are presented in Table 6.13 along with the base optimal decision vector and the optimal design estimated using the sensitivity coefficients. The response of θ_d to the parameter perturbation does not agree well with the response estimated using the sensitivity equations. The difference is due to a change in the values of the cost coefficients and exponents in the primary digester cost function. At a digester volume of 1968 m^3 the exponents in the expressions for operation and maintenance costs increase from 0.2 to 0.55. The volume of the digester equals 1549 m^3 at the base optimum and 2446 m^3 at the perturbed optimum, and the sensitivity coefficients cannot be used to estimate accurately the optimal digester volume.

6.4.4.2. Optimal Cost Sensitivity

Normalized objective sensitivity coefficients are presented in Table 6.14. Because the cost functions exhibit economies of scale with respect to unit process size, a 1% increase in Q_0 increases the total annual cost by less than 1%. The value of the objective function sensitivity coefficient, 0.607, illustrates these economies of scale. The objective sensitivities associated with the vacuum filter and sludge disposal costs are greater than the objective sensitivities associated with most other sludge treatment unit processes. These observations support the above discussion of tradeoffs within the sludge handling subsystem.

6.4.4.3. Summary

The base optimal decision variable values associated with the liquid treatment train are very robust with respect to a change in influent flow rate. A 1% increase in the influent flow rate increases the sizes of most unit processes by about 1%. The gravity thickener, the primary digester, and the secondary

Table 6.13-Compartison of Perturbation Analysis and Estimated Solution for Q_0

	Solution		
Variable	Base	Perturbation	Estimated
OR_p (m/hr)	6.00*	6.00*	6.00*
θ_c (days)	2.19	2.19	2.19
θ_{at} (days)	0.16	0.16	0.16
RR	0.13	0.12	0.12
L_{gt} (kg/day/m^2)	12.35	12.00*	11.36†
T_d (℃)	60.00*	60.00*	60.00*
θ_d (days)	14.62	14.12	15.94
L_d (kg/day/m^2)	40.17	38.41	37.64
L_f (kg/hr/m^2)	6.69	6.79	6.83

* Variable is at bound

† Estimation violates bound

digester are all more cost effective at a higher Q_0. More efficient sludge thickening and digestion units increases the net energy production from methane and damps the effect of an increase in Q_0 upon the costs associated with sludge dewatering and disposal.

6.4.5. Variation in Effluent Standards

The optimal plant design and cost is constrained by effluent limitations on BOD_5 and *TSS*. Effluent BOD_5 is composed of soluble and suspended oxygen demanding materials, and the effluent *TSS* is predicted by the Chapman model (equation 3.3). The nominal effluent standards are 30 mg/l for both BOD_5 and *TSS*.

The effluent quality of a wastewater treatment plant has a direct impact on environmental quality in the recieving water body. An important tradoff in water quality management involves the cost of waste treatment versus the damages (often unquantifiable) of environmental degradation. An analysis of the sensitivity of optimal annual cost with respect to changing effluent

Table 6.14 - Normalized objective sensitivity coefficients for Q_o

	CAPITAL	OPERATION	MAINTENANCE	MATERIAL	POWER	TOTAL
PRIMARY SETTLING TANK	$0.76995 \times 10^{+0}$	$0.29998 \times 10^{+0}$	$0.13999 \times 10^{+0}$	$0.75995 \times 10^{+0}$		$0.61369 \times 10^{+0}$
PRIMARY SLUDGE PUMPING	$0.52789 \times 10^{+0}$	$0.40837 \times 10^{+0}$	$0.42829 \times 10^{+0}$	$0.63746 \times 10^{+0}$	$0.99603 \times 10^{+0}$	$0.48502 \times 10^{+0}$
AERATION TANK	$0.71637 \times 10^{+0}$					$0.71637 \times 10^{+0}$
DIFFUSED AIR AERATION	$0.65973 \times 10^{+0}$	$0.47980 \times 10^{+0}$	$0.54977 \times 10^{+0}$			$0.60887 \times 10^{+0}$
SECONDARY SETTLING TANK	$0.76656 \times 10^{+0}$	$0.59732 \times 10^{+0}$	$0.59732 \times 10^{+0}$	$0.75660 \times 10^{+0}$		$0.72061 \times 10^{+0}$
RECYCLE SLUDGE PUMPING	$0.51975 \times 10^{+0}$	$0.98066 \times 10^{+0}$	$0.98066 \times 10^{+0}$	$0.50994 \times 10^{+0}$	$0.98066 \times 10^{+0}$	$0.65196 \times 10^{+0}$
GRAVITY THICKENER	$0.86039 \times 10^{+0}$	$0.67043 \times 10^{+0}$	$0.67043 \times 10^{+0}$	$0.84921 \times 10^{+0}$		$0.80664 \times 10^{+0}$
PRIMARY ANAEROBIC DIGESTER	$0.62311 \times 10^{+0}$	$0.21122 \times 10^{+0}$	$0.22178 \times 10^{+0}$	$0.39076 \times 10^{+0}$		$0.51831 \times 10^{+0}$
SECONDARY ANAEROBIC DIGESTER	$0.62819 \times 10^{+0}$	$0.21294 \times 10^{+0}$	$0.22359 \times 10^{+0}$	$0.39394 \times 10^{+0}$		$0.49026 \times 10^{+0}$
VACUUM FILTER	$0.67050 \times 10^{+0}$	$0.56598 \times 10^{+0}$	$0.46840 \times 10^{+0}$	$0.78118 \times 10^{+0}$		$0.67579 \times 10^{+0}$
RECYCLE STREAM PUMPING	$0.52583 \times 10^{+0}$	$0.99213 \times 10^{+0}$	$0.99213 \times 10^{+0}$	$0.00000 \times 10^{+1}$	$0.99213 \times 10^{+0}$	$0.45148 \times 10^{+0}$
FINAL SLUDGE DISPOSAL	$0.75178 \times 10^{+0}$	$0.65088 \times 10^{+0}$				$0.67135 \times 10^{+0}$
TOTALS FOR C/O/M/MAT/P	$0.68721 \times 10^{+0}$	$0.53912 \times 10^{+0}$	$0.46096 \times 10^{+0}$	$0.69134 \times 10^{+0}$	$0.98138 \times 10^{+0}$	

NET ENERGY FROM METHANE = $0.10492 \times 10^{+1}$

TOTAL OBJECTIVE SENSITIVITY = $0.60691 \times 10^{+0}$

requirements, based on a mathematical model of a wastewater treatment system, could be useful in water quality management studies. The sensitivities allow more informed decision making with respect to the desired level of effluent quality. Also, the results of a sensitivity analysis can indicate complex interactions among the unit processes, and can be used to explore trends in optimal design when effluent standards are more or less stringent.

The sensitivity analysis presented below considers improving effluent quality with respect to BOD_5 and then with respect to *TSS*. The sensitivity coefficients are first discussed. A perturbation analysis is then shown to validate the sensitivity coefficient results. Finally, normalized objective sensitivities for individual cost items are presented and used to explore the cost of increasing effluent wastewater quality.

6.4.5.1. Design Sensitivity

Normalized sensitivity coefficients of design and state variables for changes in the BOD_5 and the *TSS* effluent standards are presented in Tables 6.15 and 6.16. Optimal design of the liquid handling subsystem is quite sensitive to the values of both effluent standards, and some interesting interactions between the aeration tank and final settler are indicated.

As the *TSS* standard is tightened, removal efficiency of the final clarifier must increase. The Chapman model predicts that the effluent suspended solids will increase with increased mixed-liquor suspended solids (MLSS) concentration and increased overflow rate. Chapman's definition of the overflow rate is unconventional; the influent flow rate rather than the flow out of the clarifier is used. This accounts, in part, for the high sensitivity of the RR. Also, as effluent *TSS* decreases, the amount of soluble oxygen demanding material in the effluent can increase. Thus, a more stringent effluent *TSS* standard allows a lower solids retention time, which is accomplished by substantial decreases in the RR and the θ_{at}. The negative sensitivity coefficient of the final settler area, -.902, indicates that the decrease in RR should be accompanied by an increase in the final settler area to meet the new *TSS* standard optimally. A three-way economic tradeoff is captured by the sensitivity analysis; the cost of increasing the final settler area must be balanced with savings due to decreased recycle

Table 6.15 - Normalized sensitivity coefficients of decision and state variables for BOD_5

Variable	S_n
Design Variables	
Primary Settler Overflow Rate	$0.0000\times10^{+1}$
Solids Retention Time in Aeration Tank	$-.1259\times10^{+1}$
Hydraulic Retention Time in Aeration Tank	$-.9051\times10^{+0}$
Recycle Ratio of Activated Sludge	$-.7980\times10^{+0}$
Solids Loading on Gravity Thickener	0.3712×10^{-1}
Primary Digester Temperature	$0.0000\times10^{+1}$
Solids Retention Time in Digester	$-.1650\times10^{-1}$
Solids Loading on Secondary Digester	$-.3694\times10^{-1}$
Vacuum Filter Yield	0.1229×10^{-1}
State Variables	
Flow into Primary Settler	$-.2306\times10^{-2}$
Soluble BOD_5 Concentration into Primary Settler	0.2515×10^{-2}
Active Biomass Concentration into Primary Settler	$-.2289\times10^{-1}$
Degradable Solids Concentration into Primary Settler	0.1373×10^{-2}
Inert Solids Concentration into Primary Settler	$-.1441\times10^{-2}$
Fixed Solids Concentration into Primary Settler	$-.2603\times10^{-2}$
Total Solids Concentration into Primary Settler	$-.9253\times10^{-3}$
Area of Primary Settler	$-.2304\times10^{-2}$
Solids Removal Efficiency of Primary Settler	0.1626×10^{-3}
Flow Rate into Aeration Tank	$-.2304\times10^{-2}$
Primary Settler Underflow Flow Rate	$-.4138\times10^{-2}$
Volume of Aeration Tank	$-.9074\times10^{+0}$
Active Biomass Concentration in Aeration Tank	$-.4710\times10^{+0}$
Soluble BOD_5 Concentration out of Aeration Tank	$0.1540\times10^{+1}$
Inert to Active Microorganism Ratio in Aeration Tank	0.6136×10^{-1}
Fixed to Active Microorganism Ratio in Aeration Tank	$0.1089\times10^{+0}$
Waste sludge Ratio	$-.3403\times10^{+0}$
Combined Recycle and Waste sludge Ratios	$-.7747\times10^{+0}$
Active Biomass Concentration in Effluent	$-.4620\times10^{-1}$
Active Biomass Concentration in Return Sludge	$0.2054\times10^{+0}$
Area of Final Settler	$-.1795\times10^{+0}$
Total BOD_5 removed in Aeration Tank	$-.2184\times10^{+0}$
Diffused Air Aeration Air Flow Rate	$-.2749\times10^{+0}$
Waste Sludge Flow Rate to Gravity Thickener	$-.3426\times10^{+0}$
Return Sludge Total Solids Concentration	$0.2516\times10^{+0}$
Total Sludge Flow Rate into Gravity Thickener	$-.2971\times10^{+0}$
Soluble BOD_5 Concentration of Combined Sludge	$0.9656\times10^{+0}$
Total Solids Concentration of Combined Sludge	$0.2487\times10^{+0}$
Area of Gravity Thickener	$-.8323\times10^{-1}$

Variable	S_n
Gravity Thickener Supernatant Flow Rate	$-.4515\times10^{+0}$
Sludge Flow Rate into Primary Digester	$-.4024\times10^{-1}$
Total Solids Concentration into Primary Digester	$-.5868\times10^{-2}$
Fixed Solids Concentration into Primary Digester	0.2518×10^{-1}
$Q_7 M_{a_5} M_{t_{10}}/Q_9/M_{t_9}$	$-.8869\times10^{-1}$
$1000\, Q_8 M_{t_8} M_{t_{10}}/Q_9/M_{t_9}/M_{t_1}$	0.4592×10^{-1}
Volume of Primary Digester	$-.5674\times10^{-1}$
First Order Solids Stabilization Rate	$0.0000\times10^{+1}$
Total Heat Requirement for Digester	$-.4240\times10^{-1}$
Total Volatile Solids into Digester	$-.1757\times10^{-1}$
Inert Solids Concentration in Digester Effluent	$-.2685\times10^{-2}$
Methane Production Rate	$-.5857\times10^{-1}$
Net Energy Value of Digester	$-.6705\times10^{-1}$
Area of Secondary Digester	0.1615×10^{-1}
Flow Rate of Digester Supernatant	$0.0000\times10^{+1}$
Secondary Digester Underflow Flow Rate	$-.4024\times10^{-1}$
Total Solids Concentration in Digester Underflow	0.1944×10^{-1}
Ratio of Inert to Total Solids Concentration out of Digester	$-.2213\times10^{-1}$
Area of Vacuum Filter	$-.3086\times10^{-1}$
Vacuum Filter Supernatant Flow Rate	$-.4315\times10^{-1}$
Filter Cake Flow Rate	$-.1858\times10^{-1}$
Filter Cake Solids Concentration	$0.0000\times10^{+1}$
Mass Fraction of Primary Sludge in Combined Sludge	0.4499×10^{-1}
Combined Sludge Thickening Characteristic	0.6096×10^{-1}
Combined Sludge Thickening Characteristic	0.3626×10^{-2}
Primary Settler Underflow Solids Concentration	0.6540×10^{-3}

Table 6.16 - Normalized sensitivity coefficients of decision and state variables for *TSS*

Variable	S_n
Design Variables	
Primary Settler Overflow Rate	$0.0000 \times 10^{+1}$
Solids Retention Time in Aeration Tank	$0.4299 \times 10^{+0}$
Hydraulic Retention Time in Aeration Tank	$0.4056 \times 10^{+0}$
Recycle Ratio of Activated Sludge	$0.9039 \times 10^{+0}$
Solids Loading on Gravity Thickener	$0.1041 \times 10^{+0}$
Primary Digester Temperature	$0.0000 \times 10^{+1}$
Solids Retention Time in Digester	0.6072×10^{-3}
Solids Loading on Secondary Digester	0.3787×10^{-1}
Vacuum Filter Yield	$-.1259 \times 10^{-1}$
State Variables	
Flow into Primary Settler	0.2965×10^{-2}
Soluble BOD_5 Concentration into Primary Settler	$-.4696 \times 10^{-2}$
Active Biomass Concentration into Primary Settler	0.2429×10^{-1}
Degradable Solids Concentration into Primary Settler	$-.9691 \times 10^{-3}$
Inert Solids Concentration into Primary Settler	$-.2708 \times 10^{-2}$
Fixed Solids Concentration into Primary Settler	$-.1279 \times 10^{-1}$
Total Solids Concentration into Primary Settler	$-.3836 \times 10^{-2}$
Area of Primary Settler	0.2973×10^{-2}
Solids Removal Efficiency of Primary Settler	0.6742×10^{-3}
Flow Rate into Aeration Tank	0.2973×10^{-2}
Primary Settler Underflow Flow Rate	$-.4628 \times 10^{-2}$
Volume of Aeration Tank	$0.4086 \times 10^{+0}$
Active Biomass Concentration in Aeration Tank	0.6149×10^{-1}
Soluble BOD_5 Concentration out of Aeration Tank	$-.5256 \times 10^{+0}$
Inert to Active Microorganism Ratio in Aeration Tank	$-.2189 \times 10^{-1}$
Fixed to Active Microorganism Ratio in Aeration Tank	$-.4861 \times 10^{-1}$
Waste sludge Ratio	$0.4279 \times 10^{+0}$
Combined Recycle and Waste sludge Ratios	$0.8797 \times 10^{+0}$
Active Biomass Concentration in Effluent	$0.1019 \times 10^{+1}$
Active Biomass Concentration in Return Sludge	$-.7323 \times 10^{+0}$
Area of Final Settler	$-.9021 \times 10^{+0}$
Total BOD_5 removed in Aeration Tank	0.7187×10^{-1}
Diffused Air Aeration Air Flow Rate	0.9335×10^{-1}
Waste Sludge Flow Rate to Gravity Thickener	$0.4309 \times 10^{+0}$
Return Sludge Total Solids Concentration	$-.7516 \times 10^{+0}$
Total Sludge Flow Rate into Gravity Thickener	$0.3724 \times 10^{+0}$
Soluble BOD_5 Concentration of Combined Sludge	$-.4295 \times 10^{+0}$
Total Solids Concentration of Combined Sludge	$-.5383 \times 10^{+0}$
Area of Gravity Thickener	$-.2750 \times 10^{+0}$

Variable	S_n
Gravity Thickener Supernatant Flow Rate	$0.6980 \times 10^{+0}$
Sludge Flow Rate into Primary Digester	$-.1694 \times 10^{+0}$
Total Solids Concentration into Primary Digester	$-.1567 \times 10^{-2}$
Fixed Solids Concentration into Primary Digester	$-.2704 \times 10^{-1}$
$Q_7 M_{a_5} M_{t_{10}} / Q_9 / M_{t_9}$	$-.1356 \times 10^{+0}$
$1000 Q_8 M_{t_8} M_{t_{10}} / Q_9 / M_{t_9} / M_{t_1}$	$0.1678 \times 10^{+0}$
Volume of Primary Digester	$-.1687 \times 10^{+0}$
First Order Solids Stabilization Rate	$0.0000 \times 10^{+1}$
Total Heat Requirement for Digester	$-.1693 \times 10^{+0}$
Total Volatile Solids into Digester	0.8037×10^{-2}
Inert Solids Concentration in Digester Effluent	0.7489×10^{-2}
Methane Production Rate	$-.1616 \times 10^{+0}$
Net Energy Value of Digester	$-.1576 \times 10^{+0}$
Area of Secondary Digester	$-.2272 \times 10^{+0}$
Flow Rate of Digester Supernatant	$0.0000 \times 10^{+1}$
Secondary Digester Underflow Flow Rate	$-.1694 \times 10^{+0}$
Total Solids Concentration in Digester Underflow	$-.1993 \times 10^{-1}$
Ratio of Inert to Total Solids Concentration out of Digester	0.2742×10^{-1}
Area of Vacuum Filter	$-.1790 \times 10^{+0}$
Vacuum Filter Supernatant Flow Rate	$-.1664 \times 10^{+0}$
Filter Cake Flow Rate	$-.1916 \times 10^{+0}$
Filter Cake Solids Concentration	$0.0000 \times 10^{+1}$
Mass Fraction of Primary Sludge in Combined Sludge	$0.1639 \times 10^{+0}$
Combined Sludge Thickening Characteristic	$0.2221 \times 10^{+0}$
Combined Sludge Thickening Characteristic	0.1321×10^{-1}
Primary Settler Underflow Solids Concentration	0.2712×10^{-2}

flows and decreased hydraulic retention time. The additional driving force for decreasing the RR, due to the form of the Chapman model, pushes the optimal relationship between θ_{at} and RR towards a very low recycle flow. These complex interactions can be examined using the sensitivity analysis since the system model includes a model of the final clarifier efficiency.

The interactions discussed could be very sensitive to the type of clarifier model used. Also, the discount rate or design plant life may significantly affect the sensitivities of RR and θ_{at} since the decision about how to maintain the sludge age hinges upon capital versus operation intensive plant design.

The normalized sensitivity coefficients of plant design variables for a variation in effluent BOD_5 reveal similar interactions between the clarifier and aeration basin. As the effluent BOD_5 standard is made more stringent, the percent increase in θ_c is greater than the percent decrease in effluent BOD_5. The sensitivity coefficient of the soluble BOD_5 portion (S_3) equals 1.54. This sensitivity coefficient indicates that all the BOD_5 removed to meet a lower effluent requirement is soluble since the effluent BOD_5 for the base optimum is comprised of 2/3 soluble and 1/3 suspended material. The *TSS* standard should still be binding for small decreases in effluent BOD_5. To meet the higher sludge age, the percent increase in θ_{at} is greater than the percent increase in RR. Again, this is because of the influence RR has upon solids removal efficiency in the final clarifier. The final clarifier area should increase .179% in response to a 1% decrease in BOD_5 because of an increase in MLSS concentration in the aeration tank. It is cost effective to increase the final settler area slightly in response to a change in effluent BOD_5, rather than to keep the MLSS concentration constant through an even larger increase in θ_{at}. Increasing the final clarifier area has the additional advantage of producing thicker return sludge.

A change in either the effluent BOD_5 standard or effluent *TSS* standard affects the input to the solids handling subsystem through a change in the concentration and flow rate of the waste activated sludge. The sensitivity coefficients can be used to estimate that a decrease in the effluent *TSS* concentration *increases* the concentration and *decreases* the flow rate of the activated sludge wasted to the gravity thickener. However, a more stringent BOD_5

standard would ***decreases*** the concentration and ***increases*** the flow rate of the activated sludge sent to the solids handling subsystem. These opposite effects indicate that there are different driving forces associated with a decrease in effluent BOD_5 and with a decrease in effluent *TSS*.

The values of the sensitivity coefficients indicate that a change in either effluent standard would affect the flow rate and the concentration of the waste activated sludge, but most of the sludge treatment subsystem decision variables are relatively insensitive to small variations in the effluent BOD_5 standard or the effluent *TSS* standard. However, an interesting interaction between the liquid and solids handling subsystems exists. The increase in final clarifier area and decrease in underflow flow rate, required for optimal design at a lower effluent *TSS* concentration, create a much more concentrated combined sludge entering the gravity thickener. Less water in the sludge would yield lower heating and capital costs of the solids handling unit processes. However, the optimal design when effluent *TSS* is decreased calls for an increase in the underflow flow rate from the gravity thickener, even though the influent flow to the gravity thickener decreases. The increase in underflow flow rate causes an increase in the sizes of all sludge handling units since they must handle more sludge flow. The driving force for increasing the thickener underflow lies in the equation to predict underflow solids concentration from the gravity thickener (equation 3.2). The combined sludge settling characteristic, n_c, whose value indicates the degree of hinderance to settling due to particle interaction, equals 2.574 at the base optimum. An increase in sludge concentration would decrease the settling velocity to such an extent that increasing the underflow flow rate becomes attractive when compared to the alternative of increasing the thickener area to satisfy the solids mass balance. The sensitivity analysis has highlighted this interaction.

A perturbation analysis was performed whereby the *TSS* and BOD_5 effluent standards were individually decreased by 5 mg/l. The results of this analysis are presented in Tables 6.17 and 6.18, along with the base optimal decision variables and the decision variables estimated using the sensitivity coefficients.

The perturbed solution for a 25 mg/l BOD_5 standard validates the trends predicted by the sensitivity analysis. However, the perturbation analysis for a 25

Table 6.17-Comparison of Perturbation Analysis and Estimated Solution for BOD_5

	Solution		
Variable	Base	Perturbation	Estimated
OR_p (m/hr)	6.00*	6.00*	6.00*
θ_c (days)	2.19	2.82	2.65
θ_{at} (days)	0.16	0.19	0.18
RR	0.13	0.15	0.14
L_{gt} (kg/day/m^2)	12.35	12.34	12.27
T_d (℃)	60.00*	60.00*	60.00*
θ_d (days)	14.62	14.50	14.66
L_d (kg/day/m^2)	40.17	40.41	40.41
L_f (kg/m^2/hr)	6.69	6.68	6.68

* Variable is at bound

Table 6.18-Comparison of Perturbation Analysis and Estimated Solution for TSS

	Solution		
Variable	Base	Perturbation	Estimated
OR_p (m/hr)	6.00*	5.38	6.00*
θ_c (days)	2.19	2.05	2.04
θ_{at} (days)	0.16	0.15	0.15
RR	0.13	0.099	0.11
L_{gt} (kg/day/m^2)	12.35	12.22	12.13
T_d (℃)	60.00*	60.00*	60.00*
θ_d (days)	14.62	14.77	14.62
L_d (kg/day/m^2)	40.17	39.91	39.91
L_f (kg/m^2/hr)	6.69	6.71	6.71

* Variable is at bound

mg/l *TSS* standard illustrates a process interaction not indicated by the values of the sensitivity coefficients. At the optimum obtained, the OR_p is released from its upper bound of 6.0 m/hr. This reduces the organic load on the aeration tank and allows a lower recycle ratio to be maintained. The sludge design variables agree well with the estimated obtained using the sensitivity coefficients and the interaction of thickener influent solids concentration with the design of the other sludge handling unit processes is shown.

6.4.5.2. Optimal Cost Sensitivity

Normalized sensitivity coefficients for both *TSS* and BOD_5 effluent standards are presented in Tables 6.19 and 6.20. The total objective sensitivities can be used to estimate a perturbed optimal annual cost of $511,920 for BOD_5 and $504,813 when the *TSS* effluent standard equals 25 mg/l. These estimates compare well with the objective values from the perturbation analysis: $513,960 for BOD_5 and $505,980 for *TSS*.

Table 6.19 shows that a decrease in effluent *TSS* allows the optimal costs associated with the aeration tank, the diffused air aeration, and the recycle sludge pumping to ***decrease***. A decrease in effluent *TSS* relaxes the requirement on aerobic degradation of BOD_5 because the percentage of total effluent BOD_5 present as soluble material can increase. However, the total cost increases with a decrease in effluent *TSS* primarily because of increases in the costs associated with the final settling tank. The total annual cost must increase, otherwise the constraint on *TSS* would not be binding at the base solution.

In contrast to a decrease in the effluent *TSS* requirement, Table 6.20 shows that a decrease in effluent BOD_5 ***increases*** the optimal costs associated with aerobic removal of BOD_5 (*i.e.* aeration tank costs, diffused air aeration costs, and recycle sludge pumping costs). A decrease in effluent BOD_5 also increases the optimal costs associated with the final settling tank because the solids loading increases. However, as previously discussed, the constraint on effluent *TSS* would still be binding for a small decrease in effluent BOD_5. Thus the increases in the costs associated with the final settling tank are only enough to allow the effluent *TSS* standard to be met; the sensitivity coefficients

Table 6.19 - Normalized objective sensitivity coefficients for BOD_5

	CAPITAL	OPERATION	MAINTENANCE	MATERIAL	POWER	TOTAL
PRIMARY SETTLING TANK	$-.17743 \times 10^{-2}$	$-.69131 \times 10^{-3}$	$-.32261 \times 10^{-2}$	$-.17513 \times 10^{-2}$		$-.14142 \times 10^{-2}$
PRIMARY SLUDGE PUMPING	$-.21929 \times 10^{-2}$	$-.16964 \times 10^{-2}$	$-.17792 \times 10^{-2}$	$-.26481 \times 10^{-2}$	$-.41376 \times 10^{-2}$	$-.20148 \times 10^{-2}$
AERATION TANK	$-.64422 \times 10^{+0}$					$-.64422 \times 10^{+0}$
DIFFUSED AIR AERATION	$-.18146 \times 10^{+0}$	$-.13197 \times 10^{+0}$	$-.15121 \times 10^{+0}$			$-.16747 \times 10^{+0}$
SECONDARY SETTLING TANK	$-.13820 \times 10^{+0}$	$-.10769 \times 10^{+0}$	$-.10769 \times 10^{+0}$	$-.13641 \times 10^{+0}$		$-.12992 \times 10^{+0}$
RECYCLE SLUDGE PUMPING	$-.41178 \times 10^{+0}$	$-.77695 \times 10^{+0}$	$-.77695 \times 10^{+0}$	$-.40401 \times 10^{+0}$	$-.77695 \times 10^{+0}$	$-.51653 \times 10^{+0}$
GRAVITY THICKENER	$-.64085 \times 10^{-1}$	$-.49936 \times 10^{-1}$	$-.49936 \times 10^{-1}$	$-.63253 \times 10^{-1}$		$-.60082 \times 10^{-1}$
PRIMARY ANAEROBIC DIGESTER	$-.33476 \times 10^{-1}$	$-.11348 \times 10^{-1}$	$-.11915 \times 10^{-1}$	$-.20993 \times 10^{-1}$		$-.27846 \times 10^{-1}$
SECONDARY ANAEROBIC DIGESTER	0.95256×10^{-2}	0.32290×10^{-2}	0.33904×10^{-2}	0.59736×10^{-2}		0.74341×10^{-2}
VACUUM FILTER	$-.21912 \times 10^{-1}$	$-.10774 \times 10^{-1}$	$-.89169 \times 10^{-2}$	$-.14871 \times 10^{-1}$		$-.16629 \times 10^{-1}$
RECYCLE STREAM PUMPING	$-.16434 \times 10^{+0}$	$-.31008 \times 10^{+0}$	$-.31008 \times 10^{+0}$	$0.00000 \times 10^{+1}$	$-.31008 \times 10^{+0}$	$-.14111 \times 10^{+0}$
FINAL SLUDGE DISPOSAL	$-.14311 \times 10^{-1}$	$-.12390 \times 10^{-1}$				$-.12780 \times 10^{-1}$
TOTALS FOR C/O/M/MAT/P	$-.18094 \times 10^{+0}$	$-.43473 \times 10^{-1}$	$-.83564 \times 10^{-1}$	$-.34691 \times 10^{-1}$	$-.74656 \times 10^{+0}$	
NET ENERGY FROM METHANE =	$-.67051 \times 10^{-1}$					

TOTAL OBJECTIVE SENSITIVITY $= -.13799 \times 10^{+0}$

Table 6.20 - Normalized objective sensitivity coefficients for *TSS*

	CAPITAL	OPERATION	MAINTENANCE	MATERIAL	POWER	TOTAL
PRIMARY SETTLING TANK	0.22892×10^{-2}	0.89190×10^{-3}	0.41622×10^{-3}	0.22594×10^{-2}		0.18246×10^{-2}
PRIMARY SLUDGE PUMPING	$-.24529\times10^{-2}$	$-.18976\times10^{-2}$	$-.19901\times10^{-2}$	$-.29621\times10^{-2}$	$-.46282\times10^{-2}$	$-.22537\times10^{-2}$
AERATION TANK	$0.29008\times10^{+0}$					$0.29008\times10^{+0}$
DIFFUSED AIR AERATION	0.61610×10^{-1}	0.44807×10^{-1}	0.51342×10^{-1}			0.56861×10^{-1}
SECONDARY SETTLING TANK	$-.69464\times10^{+0}$	$-.54128\times10^{+0}$	$-.54128\times10^{+0}$	$-.68562\times10^{+0}$		$-.65301\times10^{+0}$
RECYCLE SLUDGE PUMPING	$0.46779\times10^{+0}$	$0.88263\times10^{+0}$	$0.88263\times10^{+0}$	$0.45896\times10^{+0}$	$0.88263\times10^{+0}$	$0.58679\times10^{+0}$
GRAVITY THICKENER	$-.21173\times10^{+0}$	$-.16499\times10^{+0}$	$-.16499\times10^{+0}$	$-.20898\times10^{+0}$		$-.19851\times10^{+0}$
PRIMARY ANAEROBIC DIGESTER	$-.99561\times10^{-1}$	$-.33749\times10^{-1}$	$-.35437\times10^{-1}$	$-.62437\times10^{-1}$		$-.82817\times10^{-1}$
SECONDARY ANAEROBIC DIGESTER	$-.13402\times10^{+0}$	$-.45431\times10^{-1}$	$-.47703\times10^{-1}$	$-.84048\times10^{-1}$		$-.10459\times10^{+0}$
VACUUM FILTER	$-.12706\times10^{+0}$	$-.11110\times10^{+0}$	$-.91950\times10^{-1}$	$-.15335\times10^{+0}$		$-.13078\times10^{+0}$
RECYCLE STREAM PUMPING	$0.21129\times10^{+0}$	$0.39866\times10^{+0}$	$0.39866\times10^{+0}$	$0.00000\times10^{+1}$	$0.39866\times10^{+0}$	$0.18141\times10^{+0}$
FINAL SLUDGE DISPOSAL	$-.14758\times10^{+0}$	$-.12777\times10^{+0}$				$-.13179\times10^{+0}$
TOTALS FOR C/O/M/MAT/P	$-.44389\times10^{-1}$	$-.97786\times10^{-1}$	$-.63468\times10^{-1}$	$-.14413\times10^{+0}$	$0.85048\times10^{+0}$	

NET ENERGY FROM METHANE $= -.15763\times10^{+0}$

TOTAL OBJECTIVE SENSITIVITY $= -.52769\times10^{-1}$

estimate that it is optimal to meet a more stringent BOD_5 standard by removing the additional BOD_5 aerobically.

6.4.5.3. Summary

The sensitivity analysis of the model with respect to variations in effluent standards allows the complex interactions between the aeration basin and the final settler to be shown. Because of the final clarifier model used, the recycle ratio is very sensitive to a variation in either effluent requirement, as is design of the aeration tank and the final settler area. Relative to the liquid subsystem decision variables, the optimal decision variables associated with the sludge handling unit processes are insensitive to a change in either the BOD_5 standard or the effluent TSS standard. However, the optimal costs of sludge treatment are sensitive to a change in the effluent TSS concentration. The total annual cost is more sensitive to a change in effluent BOD_5 than to a change in effluent TSS, partly because a change in the effluent TSS standard also affects the effluent BOD_5 standard.

CHAPTER 7

ROBUSTNESS CONSIDERATIONS IN WASTEWATER TREATMENT PLANT DESIGN: A PRELIMINARY ANALYSIS

7.1. Introduction

Mathematical programming techniques have been developed to obtain "optimal" solutions to many types of water resources problems, such as groundwater management, regional water allocation, and wastewater treatment plant design. The solutions obtained from optimization models of water resources systems, however, are often not optimal for the real problem for any of numerous reasons, including: (1) all of the true objectives are not known or are not quantifiable, (2) the model formulation is incomplete, and (3) the values of the model parameters are uncertain. The preliminary analysis presented in this chapter addresses the last issue: the problem of parameter uncertainty in optimization models of water resources systems. Some uncertainty in parameter values is often unavoidable and thus an important question is: How can solutions to water resources problems be obtained that are both good with respect to modeled objectives and relatively insensitive to values of some or all of the parameters?

In this chapter a method for incorporating a robustness measure into nonlinear optimization models of water resources systems is developed and investigated in a very preliminary fashion. Robustness is narrowly defined for the purpose of this research as the ability of a system design to maintain a level of system performance that meets the design criteria even if the actual values of model parameters are not exactly the same as the values assumed for design. The robustness measure to be investigated is given by the sum of the weighted sensitivities of a performance function with respect to the set or any subset of model parameter values.

The results of preliminary research to test the feasibility of a method of including a robustness performance function within a nonlinear optimization framework are presented below. The discussion describes the method and presents results obtained from applying it to the wastewater treatment plant model.

7.2. Proposed Method

A performance function for use in sensitivity analysis can be generally defined as:

$$P = h(\mathbf{x}, \boldsymbol{\theta}, \boldsymbol{\xi}) \tag{7.1}$$

That is, the performance function can be any meaningful function of the model variables and parameters. The proposed robustness measure is equal to the sum of the weighted sensitivities of P to changes in the steady state values of some or of all model parameters when $\boldsymbol{\theta}$, the set of decision variables, is fixed. Thus the following constraint is added to the original model formulation (equation (5.7)):

$$\sum_{i \in I} w_i \left| \frac{\partial P}{\partial \xi_i} \right| \leq \delta \tag{7.2}$$

where w_i is a constant weight associated with parameter ξ_i, δ is an allowable tolerance for the sum of the weighted sensitivities, and I is the set of model parameter indices, i, selected for inclusion in the robustness performance measure. The rationale behind the above constraint formulation is that it is desirable that the design or management strategy associated with a water resources project (*i.e.* $\boldsymbol{\theta}$ is fixed) be insensitive to changes in those parameter values that are uncertain, given a suitable performance function, P. The values of the weights, w_i, should be chosen to reflect estimated ranges of the parameter values since $\Delta\xi_i \left| \frac{\partial P}{\partial \xi_i} \right|$ is a linear estimate of the magnitude of the change in P due to a change in ξ_i of magnitude $\Delta\xi_i$. Note that w_i could be based on any available statistic of the estimated probability distribution of the parameter ξ_i (*e.g.* the standard deviation).

The constraint given by equation (7.2) can be included in an optimization framework. Multiobjective optimization techniques can then be used to quantify the important tradeoff between the modeled objective and the robustness measure. The value of the robustness measure can be computed from the sensitivity equations due to Chang [1967] (see section 5.4.1.).

From equation (5.8) (when J is replaced by P) the above constraint can be written:

$$\sum_{i \in I} w_i \left| \sum_{j=1}^{s} \frac{\partial h}{\partial x_j} \frac{\partial x_j}{\partial \xi_i} + \frac{\partial h}{\partial \xi_i} \right| \leq \delta \tag{7.3}$$

The sensitivities of the state variables with respect to a change in ξ_i, $\dfrac{\partial x_j}{\partial \xi_i}$ $j=1,2,\ldots,s$, are calculated by differentiating the constraint set with respect to ξ_i as in equation (5.9). From equation (5.10) the robustness constraint given by equation (7.3) can be written in vector matrix notation as:

$$\sum_{i \in I} w_i \left| -\left\{\frac{\partial h}{\partial \mathbf{x}}\right\}^T \left[\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]^{-1} \left\{\frac{\partial \mathbf{g}}{\partial \xi_i}\right\} + \frac{\partial h}{\partial \xi_i} \right| \leq \delta \tag{7.4}$$

In the above equation { } denotes a column vector, [ ] denotes a square matrix, T denotes the transpose, and $[\ ]^{-1}$ denotes the inverse of a matrix. All derivatives are evaluated at a set of decisions, $\boldsymbol{\theta}$, and at the nominal parameter values. The optimization model to be solved after inclusion of the robustness performance measure is:

$$\textit{Minimize } J = f(\mathbf{x},\boldsymbol{\theta},\boldsymbol{\xi}) \tag{7.5}$$

$$\textit{Subject to: } g_k(\mathbf{x},\boldsymbol{\theta},\boldsymbol{\xi}) = 0 \qquad k = 1,2,\ldots,s$$

$$\sum_{i \in I} w_i \left| -\left\{\frac{\partial h}{\partial \mathbf{x}}\right\}^T \left[\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]^{-1} \left\{\frac{\partial \mathbf{g}}{\partial \xi_i}\right\} + \frac{\partial h}{\partial \xi_i} \right| \leq \delta$$

Given values of $\mathbf{x}$, $\boldsymbol{\theta}$, and $\boldsymbol{\xi}$ one possible algorithm (in a preliminary stage of development) for calculating the robustness constraint in the above optimization model is:

initialization step:

Set p = the number of model parameters, i = 0, and R = the value of the robustness constraint = 0

step 1:

Calculate the constraint gradient matrix $\left[\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]$, and the vectors of partial derivatives of the performance function P, $\left\{\frac{\partial h}{\partial \mathbf{x}}\right\}$, and $\left\{\frac{\partial h}{\partial \xi_i}\right\}_{i \in I}$

step 2 (optional):

Calculate diagonal scaling matricies, $\mathbf{D}_1$ and $\mathbf{D}_2$ for scaling the linear equations and unknowns

step 3:

set $i = i+1$; if $i \in I$ then compute $\left\{\frac{\partial \mathbf{g}}{\partial \xi_i}\right\}$, otherwise go to step 6

step 4:

solve the linear system of equations $\left[\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]\left\{\frac{\partial \mathbf{x}}{\partial \xi_i}\right\} = \left\{\frac{-\partial \mathbf{g}}{\partial \xi_i}\right\}$ or (optional) the scaled system $\left(\mathbf{D}_1^{-1}\left[\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]\mathbf{D}_2\right)\{\mathbf{y}\} = \mathbf{D}_1^{-1}\left\{\frac{-\partial \mathbf{g}}{\partial \xi_i}\right\}$ for the unknowns $\left\{\frac{\partial \mathbf{x}}{\partial \xi_i}\right\}$ or $\{\mathbf{y}\}$

step 5:

Calculate:

$$R = R + w_i \left| \left\{\frac{\partial h}{\partial \mathbf{x}}\right\}^T \left\{\frac{\partial \mathbf{x}}{\partial \xi_i}\right\} + \frac{\partial h}{\partial \xi_i} \right|$$

or (optional):

$$R = R + w_i \left| \left\{\frac{\partial h}{\partial \mathbf{x}}\right\}^T \mathbf{D}_2\{\mathbf{y}\} + \frac{\partial h}{\partial \xi_i} \right|$$

step 6:

if $i<p$, go to step 3. Otherwise, stop.

Since the robustness constraint given in equation (7.4) is a function of the decision variables, incorporating the above algorithm into a nonlinear optimization framework involves performing steps 1 through 6 *each time the values of the constraints are required.* In addition, if a gradient optimization method is used, the gradient of the robustness constraint must be evaluated numerically (*i.e.* using finite difference techniques).

7.3. Discussion of the Proposed Method

An advantage of the proposed robustness measure is that it is conceptually simple, and that development and investigation of the technique should allow its straightforward application to other models of water resources systems. Optimization of the general system given by equation (7.5) should yield solutions that are good with respect to the modeled objective, J, and for which the value of the performance function, P, will change relatively little if the actual values of model parameters are different from those values assumed for design (*i.e.* the system is robust in this respect). The characteristics of the robust solutions can then be examined to develop new insights about the problem being considered. The main purpose of using a robustness constraint should be to obtain such insights; the primary emphasis should not be on the detailed numerical results. This use of optimization models is consistent with their role in the solution of complex water resources problems: to aid in the selection of a few "good" alternatives and in the understanding of these alternatives and their impacts [Brill et al., 1981, Liebman, 1976]

The robustness measure to be developed and investigated does not have a statistical basis. No notions of the probability of being in one state or another, or of the traditional statistical definitions of confidence interval or tolerance are included in the constraint formulation given by equation (7.4). As suggested in the next subsection, however, the approach should provide useful insights even if statistical information is not used.

The proposed approach does not, of course, apply to all issues related to the robustness of water resources systems. For instance, the robustness measure to be developed does not deal with unmodeled economic and political factors that may be important in the decision making process, nor does it consider the robustness of decisions to alternative model formulations (see one discussion of these issues by Fiering [1976]). Also, since the sensitivity coefficients are derived for a single parameter change, the robustness measure is heuristic in the sense that it neglects interdependent effects.

7.4. Preliminary Results of an Example Wastewater Treatment Plant Design Problem

One preliminary set of optimization analyses has been carried out using the wastewater treatment plant model to illustrate the ideas discussed above. The designers of wastewater treatment plants would be concerned if, for a particular design, the effluent water quality is sensitive to parameter values. This sensitivity should provide a meaningful robustness measure, and therefore a simple but logical performance function, P, for plant design is the weighted sum of BOD_5 and TSS in the effluent, which can be expressed as:

$$P = \alpha BOD_5 + (1 - \alpha) TSS \tag{7.6}$$

where the value of alpha is in the interval [0,1].

A preliminary computer code was developed for implementing the algorithm in section 7.2. and for incorportating it into a generalized reduced gradient nonlinear optimization package [GRG2, Lasdon et al., 1979]. Note that the robustness constraint cannot be simply added to the initial model formulation but must be evaluated as described in section 7.2 every time the values of the constraints are required. The following assumptions were made in determining the preliminary results presented below:

[1] $I = \{Q_0, M_{d_0}, S_0\}$ where Q_0, M_{d_0}, and S_0 are the influent flowrate, the influent degradable solids concentration, and the influent soluble BOD_5 concentration, respectively.

[2] $\alpha = 0.5$ (equation 7.6).

[3] The $w_i's$ associated with the parameters Q_0, M_{d_0}, and S_0 were arbitrarily chosen to equal the nominal values of the parameters (Table 3.2) (this implies that the robustness constraint is the sum of the normalized sensitivity coefficients given by equation 5.6).

[4] The effluent standards were set at 30 mg/l for both BOD_5 and *TSS*.

The right hand side of the robustness constraint, δ, was varied to explore the tradeoff with annual cost. The least cost solution corresponding to each value of δ is shown in Figure 7.1 along with the effluent quality associated with each decision vector. The annual costs shown span the range from \$500,000, associated with the least cost plant without a constraint on robustness, to \$600,000, representing approximately a 20% increase. For this increase in annual cost, the robustness measure decreases from 1.48 to 0.82, approximately a 45% decrease. This decrease in the robustness measure represents a decrease in sensitivity of the effluent water quality to changes in the steady state values of Q_0, M_{d_0}, and S_0. In this example there obviously exists a significant tradeoff between cost efficiency and this robustness measure. Also, as is apparent from Figure 7.1, both effluent BOD_5 and effluent *TSS* decrease with a decrease in the robustness measure.

Figure 7.2 shows the fractional change in some secondary treatment variables vs. the value of the robustness measure used for the example problem. The fractional change is obtained by dividing a variable value, x_j, by its respective value obtained when minimizing cost without a constraint on robustness, x_j^*. Table 7.1 compares the least cost design with the "most robust" design shown in Figure 7.2 (associated with the lowest value of the robustness measure), and compares both the least cost and the robust design with recommended design variable values.

Preliminary results indicate that significant design changes are required when the robustness of the design is accounted for in the optimization model. The robust designs exhibit marked increases in the sludge age, the recycle ratio, the hydraulic retention time in the aeration tank, and the area of the final settling tank. These trends in robust design make sense, as they are consistent

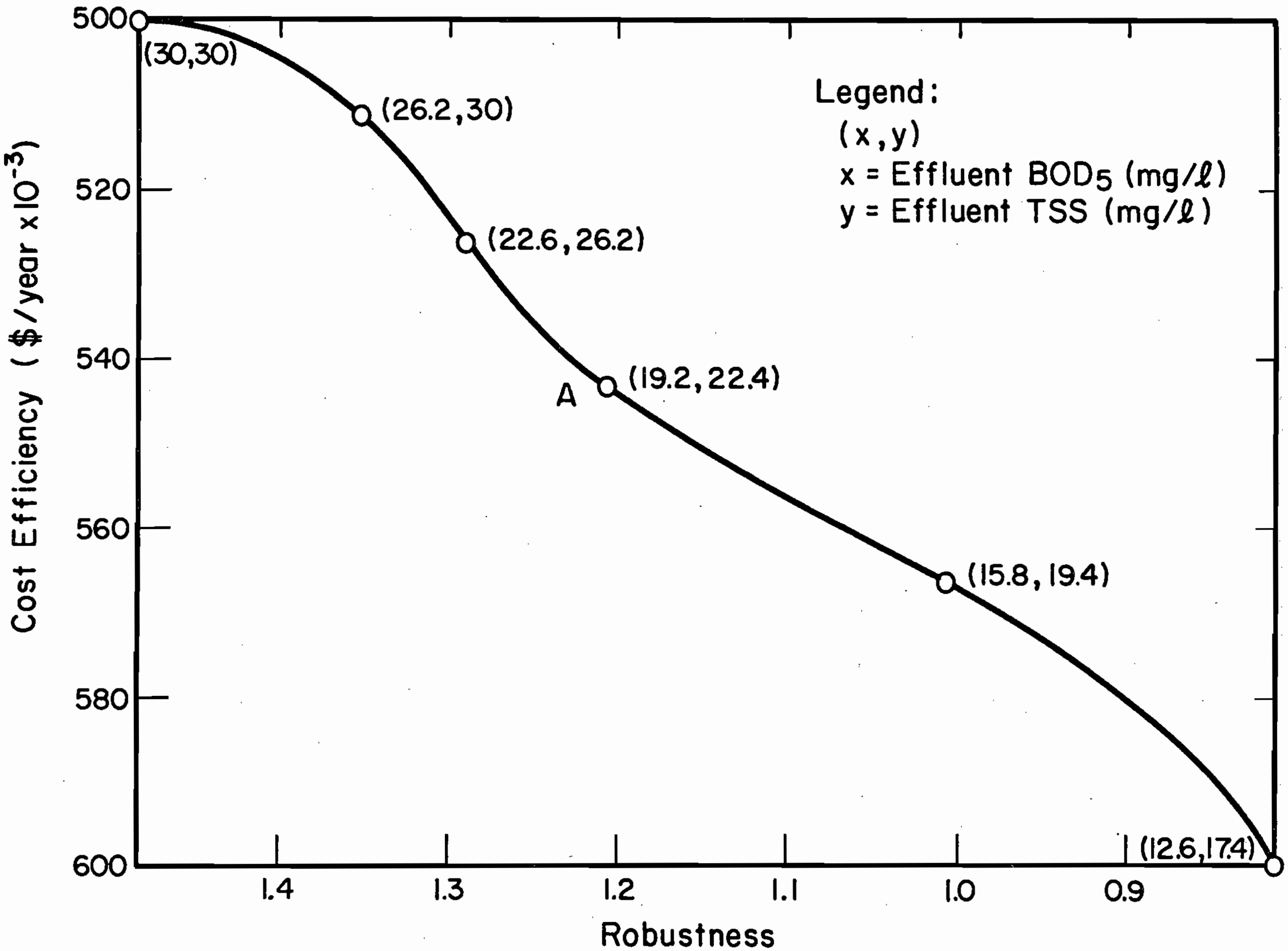

Figure 7.1 — Total System Cost vs. Robustness for $I = \{Q_o, M_{d_o}, S_o\}$, $\alpha = 0.5$, $w_i = \frac{\xi_i}{P}$

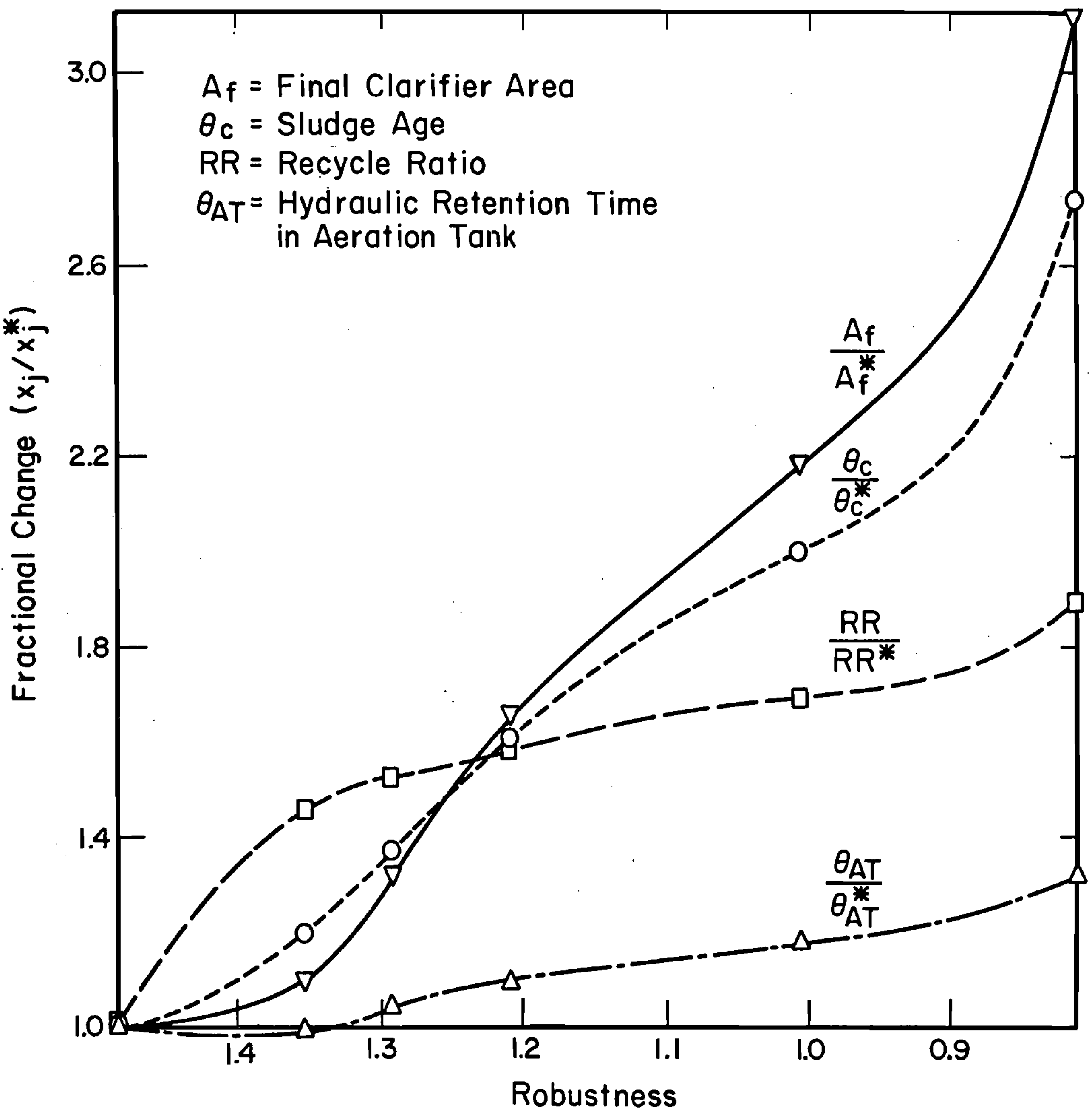

Figure 7.2 — Fractional Change in Secondary Treatment Variables vs. Robustness for $I = \{Q_o,\ M_{d_o},\ S_o\}$, $\alpha = 0.5$, $w_i = \frac{\xi_i}{P}$

Table 7.1 - Comparison of Least Cost and Robust Designs

Design Parameter	Least Cost	Robust	Recommended†
Sludge Age (days)	2.2	6.0*	5.0-15.0
Mixed Liquor Suspended Solids (g/m^3)	1520	3100	1500-3000
Hydraulic Retention Time (hrs.)	3.75	5.0	4.0-8.0
Recycle Ratio (%)	12.5	24.0	25.0-50.0
Volume Loading on Aeration Tank $\left(\frac{\text{kg}BOD_5}{\text{m}^3\text{- day}}\right)$	1.0	0.76	0.3-0.6
Overflow Rate of Final Settler (m/hr)	2.19	0.7	0.67-1.33
Solids Loading on Final Settler $\left(\frac{\text{kg}}{\text{m}^2\text{- hr}}\right)$	3.32	2.72	3.0-6.0

† Metcalf and Eddy, Inc. [1979]

* Variable is at Upper Bound

with those designs observed to work in practice. However, the trends in robust design are in ***direct conflict*** with the optimal decisions obtained when minimizing annual cost without a constraint on robustness. This analysis has illuminated in a very preliminary fashion tradeoffs between cost and robustness which may exist in the design of an activated sludge wastewater treatment plant.

Although the idea of using optimization models that include a constraint on system sensitivity is not new, research on the practical application of this idea to complex water resources problems is needed. The method illustrated above permits the tradeoff between modeled objectives and a robustness measure to be quantified in a heuristic fashion. For many complex water resources issues the nature of this tradeoff is not intuitive and is not practical to determine by other methods. Thus, further development of the proposed technique should make possible new insights about many complex water resources problems.

CHAPTER 8

SUMMARY AND FUTURE RESEARCH

8.1. Summary

This research uses a mathematical wastewater treatment plant model, developed by Tang [1984], to illustrate a method of sensitivity analysis in non-linear programming (NLP). A sensitivity equation approach is used to calculate normalized sensitivity coefficients, which approximate the percent changes in model variables and objective function due to a small parameter variation. Information from sensitivity coefficients provide some general insights into the quality of the model and sensitivity of the base solution. The discussion of selected parameter variations is summarized below.

[1] The OR_p is very insensitive and remains at its upper bound, except when the *TSS* effluent constraint is tightened.

[2] In the activated sludge system, θ_c is very insensitive to changes in all parameters other than effluent conditions. This is due to effluent quality requirements and use of a final clarifier model which predicts decreased solids removal efficiency with an increased MLSS concentration.

[3] A significant interaction exists between the RR, θ_{at}, and the area of the final settler. The area of the final settler is important in meeting the *TSS* standard, and the RR and θ_{at} are important in maintaining θ_c. The interaction is due to the presence of the recycle mass balances and the clarifier efficiency model. The area of the clarifier affects RR and θ_{at} through the thickening equation, and the RR and θ_{at} affect the clarifier area by reducing or increasing the solids removal efficiency.

[4] It is very important to maintain a high level of solids destruction in the digester. The decision to heat the digester to the upper bound on T_d is very robust; not one parameter variation considered produced a change in the value of T_d. Similarly, θ_d is very sensitive to parameter variations which affect the digester influent conditions or stabilization efficiency. The high costs associated with dewatering and disposal of the digested sludge are partially responsible for the sensitivity of θ_d.

[5] A major tradeoff within the sludge handling subsystem is between the costs of mechanical dewatering and landfilling the sludge cake versus costs of gravity thickening and sludge stabilization.

[6] Although the loadings are sensitive to some parameter variations, the decisions to build a large gravity thickener and small secondary digester are optimal for all parameter changes considered. This is due to the heating costs in the primary digester and the relatively poor settling qualities of digested sludge.

[7] For all parameter variations considered, the effects estimated using the sensitivity coefficients and validated by perturbation analyses can be explained by drawing upon practical knowledge of wastewater treatment. This result is significant since it shows that the model captures many important process interactions.

The sensitivity equations are shown to be useful in NLP sensitivity analysis. Among their important attributes are:

[1] The sensitivity equations produce much useful information for little computational expense. To obtain sensitivity coefficients for all model variables and annual cost, with respect to every model parameter, required approximately \$3 in computing charges. By comparison, a complete perturbation analysis would require an estimated \$1000 in computing charges. This comparison is based on batch rates on the CYBER system at the University of Illinois, and on allowing one 60 CPU second optimization run per parameter perturbatlion.

[2] The normalized sensitivity coefficients provide a convenient way to screen large numbers of parameter effects.

[3] The sensitivity equations, because of the economy and form of the results, could be an important aid to the modeling process. This method might be applicable to an interactive framework in which the modeler may make changes and observe the sensitivity results.

[4] The values of the sensitivity coefficients can indicate important interactions which exist among the model equations. Although not shown in this work, the sensitivity coefficients can also be used to highlight deficiencies in the model formulation or in the model equations.

[5] In many cases, the sensitivity coefficients can be used to estimate accurately design changes and objective function changes that would result from a small parameter variation.

The analysis of robustness in wastewater treatment plant design presented in chapter 7 illuminates in a very preliminary fashion tradeoffs between cost and robustness which may exist in the design of activated sludge wastewater treatment plants. The trends in robust design are in direct conflict with the optimal decisions obtained when minimizing annual cost without a constraint on robustness. Furthermore, these trends in robust design make sense, as they are consistent with those designs observed to work in practice.

8.2. Future Research

This sensitivity analysis of the wastewater treatment plant is far from complete. To investigate the model thoroughly additional analysis is needed:

[1] The sensitivity to different process models, not just parameter changes, should be examined.

[2] Sensitivity coefficients should be obtained for several optimal solutions, corresponding to different sets of plant influent and effluent conditions. This analysis would indicate how robust the results are to changes in plant size and efficiency and in wastewater characteristics.

[3] In reality, many parameters may change simultaneously; parameters thought to interact should be varied simultaneously so that these effects can be examined.

The expression for the objective function sensitivity coefficient derived by Chen et al. [1970] is different from the one used in this work, and the two expressions have been shown to yield different results for one simple example problem. Further research is needed to investigate the limitations of each approach.

The widespread use of sensitivity equations for post-optimality analysis and as an aid to model building depends on the availability of easy-to-use computer software. Some desireable features of such software are:

[1] Availability of an interface allowing the software to drive several popular NLP codes, such as GRG2. This software interface would eliminate the separation between optimization and sensitivity analysis and would accomodate an interactive framework.

[2] Ability to compute lagrange multipliers automatically, should the optimization code used not provide them.

[3] Generation of the coefficient matrix using numerical approximations of the derivatives, or using user supplied analytical expressions.

[4] Ability to approximate the new decision vector and optimal cost associated with a small parameter change.

[5] Ability to display results graphically or in a tabular format.

[6] Inclusion of simple sorting routines, allowing the user to view the 10 largest sensitivity coefficients, for example.

[7] Ability to generate code compatible with the optimization routine for inclusion of sensitivity constraints, allowing the user to evaluate the tradeoff between a specified objective and parameter sensitivity.

[8] For large problems, the ability to run the optimization and desired sensitivity analyses in a batch mode, thereby freeing the user from unnecessary time at a terminal.

Further research is needed to develop and investigate the usefulness of the method for incorporating a robustness constraint into nonlinear optimization models of water resources systems. The preliminary analysis presented in chapter 7 shows that the robustness constraint permits the tradeoff between modeled objectives and a robustness measure to be quantified in a heuristic fashion. For many complex water resources issues the nature of this tradeoff is not intuitive and is not practical to obtain by other methods. Thus, further development of the technique should make possible new insights about many complex water resources problems.

The wastewater treatment plant design problem could be used to further develop and illustrate the mathematical approach to robustness outlined in chapter 7. This problem is an important one since billions of dollars are being spent each year in the United States to construct and operate wastewater treatment plants. Furthermore, numerous concerns have recently been expressed about inadequate levels of performance of many plants. Thus, insights obtained from additional research on robustness issues associated with wastewater treatment plant design would be very timely.

APPENDIX A

MODEL VARIABLES AND SYMBOLS

Variable	Symbol
Design Variables-	
Primary Settler Overflow Rate	OR_p
Solids Retention Time in Aeration Tank	θ_c
Hydraulic Retention Time in Aeration Tank	θ_{at}
Recycle Ratio of Activated Sludge	RR
Solids Loading on Gravity Thickener	L_{gt}
Primary Digester Temperature	T_d
Solids Retention Time in Digester	θ_d
Solids Loading on Secondary Digester	L_d
Vacuum Filter Yield	L_f
State Variables	
Flow into Primary Settler	Q_1
Soluble BOD_5 Concentration into Primary Settler	S_1
Active Biomass Concentration into Primary Settler	M_{a_1}
Degradable Solids Concentration into Primary Settler	M_{d_1}
Inert Solids Concentration into Primary Settler	M_{i_1}
Fixed Solids Concentration into Primary Settler	M_{f_1}
Total Solids Concentration into Primary Settler	M_{t_1}
Area of Primary Settler	A_p
Solids Removal Efficiency of Primary Settler	M_{t_2}/M_{t_1}
Flow Rate into Aeration Tank	Q_2
Primary Settler Underflow Flow Rate	Q_8
Volume of Aeration Tank	V_{at}
Active Biomass Concentration in Aeration Tank	M_{a_3}
Soluble BOD_5 Concentration out of Aeration Tank	S_3
Inert to Active Microorganism Ratio in Aeration Tank	M_{i_3}/M_{a_3}
Fixed to Active Microorganism Ratio in Aeration Tank	M_{f_3}/M_{a_3}
Waste sludge Ratio	w
Combined Recycle and Waste sludge Ratios	$RR + w$

Variable	Symbol
Active Biomass Concentration in Effluent	M_{a_4}
Active Biomass Concentration in Return Sludge	M_{a_5}
Area of Final Settler	A_f
Total BOD_5 removed in Aeration Tank	S
Diffused Air Aeration Air Flow Rate	Q_a
Waste Sludge Flow Rate to Gravity Thickener	Q_7
Return Sludge Total Solids Concentration	M_{t_7}
Total Sludge Flow Rate into Gravity Thickener	Q_9
Total Solids Concentration of Combined Sludge	M_{t_9}
Area of Gravity Thickener	A_g
Gravity Thickener Supernatant Flow Rate	Q_{10}
Sludge Flow Rate into Primary Digester	Q_{11}
Total Solids Concentration into Primary Digester	$M_{t_{11}}$
Fixed Solids Concentration into Primary Digester	$M_{f_{11}}$
$Q_7 M_{a_5} M_{t_{10}} / Q_9 / M_{t_9}$	--
$1000\, Q_8 M_{t_8} M_{t_{10}} / Q_9 / M_{t_9} / M_{t_1}$	--
Volume of Primary Digester	V_d
First Order Solids Stabilization Rate	K_1
Total Heat Requirement for Digester	q
Total Volatile Solids into Digester	$M_{a_{11}}+M_{d_{11}}+M_{i_{11}}$
Inert Solids Concentration in Digester Effluent	$M_{i_{11}}$
Methane Production Rate	G
Net Energy Value of Digester	N
Area of Secondary Digester	A_d
Flow Rate of Digester Supernatant	Q_{13}
Secondary Digester Underflow Flow Rate	Q_{14}
Total Solids Concentration in Digester Underflow	$M_{t_{14}}$
Ratio of Inert to Total Solids Concentration out of Digester	$M_{i_{11}}/M_{t_{11}}$
Area of Vacuum Filter	A_v
Vacuum Filter Supernatant Flow Rate	Q_{15}
Filter Cake Flow Rate	Q_{16}
Filter Cake Solids Concentration	$M_{t_{16}}$
Mass Fraction of Primary Sludge in Combined Sludge	f_p
Combined Sludge Thickening Characteristic	a_c
Combined Sludge Thickening Characteristic	n_c
Primary Settler Underflow Solids Concentration	M_{t_8}

APPENDIX B

OPTIMIZATION RESULTS FOR BASE PLANT CONDITIONS USING SEVERAL STARTING POINTS

Variable		Starting Point 1	Starting Point 2	Starting Point 3	Starting Point 4
OR_p (m/hr)	intial value	0.66	0.67	1.72	6.00†
	final value	6.0†	6.00†	5.54	6.00†
θ_c (days)	initial value	5.00	4.00	3.00	2.00
	final value	2.19	2.19	2.19	2.19
θ_{at} (days)	initial value	0.32	0.10*	0.10*	0.20
	final value	0.16	0.16	0.16	0.16
RR	initial value	0.041	0.54	0.29	0.11
	final value	0.13	0.13	0.12	0.13
L_g (kg/day/m^2)	initial value	12.00*	34.51	48.00†	12.00*
	final value	12.30	12.34	12.18	12.29
T_d ($^\circ C$)	initial value	41.88	20.00*	20.00*	60.00†
	final value	60.00†	60.00†	60.00†	60.00†
θ_d (days)	initial value	30.00†	30.00†	28.33	23.00
	final value	14.80	14.63	14.90	14.80
L_d (kg/day/m^2)	initial value	12.00*	12.00*	48.00†	43.20
	final value	40.19	40.16	39.48	40.13
L_f (kg/hr/m^2)	initial value	17.18	10.14	6.31	6.50
	final value	6.71	6.69	6.73	6.69
Total Cost ($/year)	initial value	784,667	831,250	772,905	541,267
	final value	502,061	500,390	500,727	500,401
CPU seconds		416.93	611.35	473.13	249.29

† value of variable at its upper bound

* value of variable at its lower bound

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